The Role of Pass-Through Contracts in Environments with Volatile Input Prices and Frictions

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Abstract

We model a bilateral supply chain with stochastic demand, stochastic input costs, production lead times, and working capital constraints. The supply chain participants contract as follows: Either they use the pass-through contract under which the upstream supplier passes her entire commodity input cost onto the downstream assembler, or they use an appropriately adapted revenue sharing contract under which the firms split both the production costs and the operating revenues. In the absence of financing needs for either firm, the pass-through contract is dominated by the revenue sharing contract – even if downstream buyer hedges all input costs. However, when working capital limitations drive financing needs in the chain, the financial frictions break the coordinating nature of the revenue sharing contract, and the created double marginalization inefficiencies and financing costs for firms with differential working capital and financing needs weaken the profit performance of the contract. Pass-through contracts do dominate revenue sharing ones when there are low (or no) working capital suppliers. Hedging behavior can be justified even in the absence of financing frictions for pass-through contracts, and it only involves the buyer. Hedging behavior in revenue sharing contracts happens when financing is needed, and either firms both hedge, or neither hedges, all commodity purchases in the supply chain. Double marginalization inefficiencies versus financing costs are the main factors in determining the effectiveness of the contracts, with financing cost dominated environments favoring the pass-through contract.
1 Introduction

Much of the extant supply chain literature takes production costs as deterministic. However, most industrial products companies consume commodities such as raw materials and energy to power their manufacturing processes and the prices they have to pay for these commodities are hard to pin down because of the uncertainty inherent in commodity markets. To illustrate, from January 2012 to May 2013, the prices of steel billets on the London Metal Exchange dropped from $450 to $120 per ton. They then rose back up to $480 over the next 15 months. In January 2016, the price was around $170 per ton (Source: https://www.lme.com).

Given its prevalence, one might guess that the topic of managing fluctuating prices of raw materials would command a great deal of attention in the literature and that managers would therefore have sufficient guidance from which to draw in designing their risk management strategies. Such a guess would, however, be only partially correct. Although, the research literature has put forth many recommendations as to when managers should pay attention to risk, much of this information applies to standalone firms (e.g., see Smith and Stulz, 1985; Froot et al., 1993).

A commonly given recommendation in commodity price risk supply chains is that vulnerable suppliers can inexpensively manage price risk by passing it downstream and thereby avoid the risk altogether (e.g., Hartley and Zsidisin, 2012, §5). This type of arrangement is commonly referred to as the pass-through contract and there is evidence of its widespread use in practice (Matthews, 2011). The logic behind the pass-through is that larger, more sophisticated, downstream buyers can easily absorb any commodity price risk by either adjusting the selling price of their output or by hedging.

Surprisingly, however, the pass-through contract has received little or no attention in the academic literature and as such its strengths and weaknesses are not yet well understood. The goal of this paper is to take the first step towards filling this gap and to answer the following questions: (Q1) Under what conditions should firms use the pass-through contract? (Q2) When compared to the pass-through contract, can we identify another, simple contract that would make everyone in the supply chain better off? (To answer this question, we put the pass-through contract against the well-known revenue sharing contract.) (Q3) How does the choice of the supply chain contract affect how firms hedge their stochastic input costs (if at all)?

1Definition. Throughout the paper, to “hedge” means to buy or sell commodity futures as a protection against loss or failure due to price fluctuation. For details, see Van Mieghem, 2003, p.271.
To answer these questions, we use a stylized model of a bilateral supply chain where the upstream firm (a component supplier) faces stochastic production costs, due to uncertain prices of his commodity inputs, and the downstream firm (a final goods assembler) faces both stochastic production costs, due to her direct purchases of commodity inputs, and stochastic demand. Both firms are risk neutral and have limited working capital. With these assumptions, we consider two supply chain contracts: (a) a pass-through contract and (b) a revenue sharing contract.

We first analyze a benchmark case of our model where working capital limitations and need for financing commodity input purchases are not an issue. For this case, our analysis reveals that as a risk protection for the upstream supplier the pass-through contract disappoints because the downstream assembler is rational and defaults on the pass-through contract whenever her operating costs sufficiently exceed her operating revenues. Although the assembler could guarantee the pass-through performance by hedging she does not want to hedge because hedging reduces her expected payoff.

In contrast, in the no borrowing benchmark case, the revenue sharing contract achieves first-best and can arbitrarily allocate the integrated supply chain payoff between the two firms. Neither of the two firms has any incentives to hedge under this contract. Since coordination is something that the pass-through contract cannot achieve, it is easy to establish that the revenue sharing contract dominates the pass-through one.

Our model then considers an environment where each firm’s limited working capital necessitates borrowing to purchase their commodity inputs and to produce. In this new environment, the pass-through contract often has the downstream player hedging to reduce her financing costs and offer a risk-free commodity input environment for her supplier, thus reducing his financing costs as well. For the revenue sharing contract, financial frictions may lead to one of two things to happen: either the stochastic input costs cause the revenue sharing contract to be infeasible to implement due to one, or both, of the players being credit rationed; or the revenue sharing contract looses efficiency, and is no more coordinating, due to financial friction costs not being allocated effectively between firms with different working capitals and financing needs. In fact, in this new environment the efficiency loss can be so significant that it becomes possible for some cases the pass-through contract to dominate the revenue sharing contract. Environments for which the pass-through contracts prove to be effective are characterized by low (or no) working capital suppliers, supplier expectations for high profits (or high margins), and reasonable working capital availability buyers.
With respect to hedging, we find that the nature of the hedging policy is heavily influenced by the strategic interactions of the two firms. For the pass-through contract the equilibrium hedging policy is asymmetric, in that only the downstream firm hedges on behalf of the supply chain. The hedging behavior of the upstream firm is irrelevant. In the no borrowing case, the assembler hedges only when the upstream supplier requires a guaranteed level of payoff as a pre-requisite to entering into the pass-through contract; otherwise the assembler has no incentive to hedge. With borrowing, the assembler has an added incentive to hedge to reduce her expected financing costs – interestingly, if the downstream assembler hedges, then the expected financing costs of the upstream supplier disappear altogether.

With revenue sharing contract, firms also hedge to reduce their expected financing costs (there is no hedging in the no borrowing model). The nature of the hedging policy, however, is different from the pass-through as either both firms hedge both commodities, or neither firm hedges.

The rest of the paper is organized as follows. In what follows, we review related literature, present the model (§3) and derive the firms’ equilibrium behavior with the pass-through and revenue sharing in the no borrowing case (§5) and then the borrowing case (§6). §7 concludes.

2 Literature Review

In order to provide rationale for hedging, extant theories have relied on the existence of taxes (e.g., Smith and Stulz, 1985), asymmetric information (e.g., DeMarzo and Duffie, 1991), and costly external capital (e.g., Froot et al., 1993). All these papers, however, take a firm as the basic “unit of analysis.” That is, cash flows under alternative hedging scenarios are exogenously specified and the firm’s problem is to choose that hedging strategy which maximizes its expected payoff. Building on the insights from these hedging theories, many papers take the decision to hedge “as given” and provides insights into how to hedge (Neuberger, 1999).

In contrast to the corporate finance literature, where hedging is mainly about managing financial frictions, the operations management literature studies settings where uncertainty is the bane of operations and takes hedging as one of the many possible coping strategies firms can employ. More recent and influential papers in this area include Gaur and Seshadri (2005) and Caldentey and Haugh (2009). Gaur and Seshadri (2005) address the problem of using market instruments to hedge stochastic demand. Caldentey and Haugh (2009) add to these results by showing that financial hedging of
stochastic demand can lead to an increase in a budget-constrained firm’s output.

Other papers in the operations management literature discuss and analyze hedging strategies for managing risky supply (rather than risky demand) and ask whether or not it makes a difference if firms hedge risks financially or operationally. While intuition may suggest that operational and financial hedging may be substitutes, surprisingly, Chod et al. (2010) show that they can be complements. Van Mieghem (2003) offers examples of operational hedges (excess capacity, inventory, dual sourcing, etc.) Supplier subsidy (e.g., Babich, 2010) is an important example of a financial strategy designed to reduce the likelihood of supply contract abandonment.

Theoretical studies of firms’ hedging policies in bilateral supply chains has not received much attention in the literature. Stulz (1996) argues that hedging is important in supplier buyer relationships, it can reduce the risk of financial distress, and hence reduce possible switching costs between trading partners. Kang et al. (2012) empirically investigate the impact of supplier-buyer relationship on the suppliers hedging policy. Supplier hedging allows suppliers to receive more favorable contract terms from their buyers, lower expected switching costs in case of a relationship breakup, and reduce buyers concerns about potential liquidation risk. Comparing to the above papers, our work captures the strategic interaction of the firms in the supply chain in the presence of stochastic input costs and under specific supply chain contracts. Our emphasis is on how different contracts perform in the presence of financial frictions and stochastic costs, with hedging applied as appropriately to complement operational decisions.

The work closest to ours is the study of Turcic et al. (2015), which also builds the model of an uncertain demand bilateral supply chain with risk neutral firms in the presence of stochastic costs. The supply chain is operated under a wholesale price contract. The focus of the work is on explaining the rationale of why in a frictionless and risk neutral supply chain setting hedging policies might make sense and create value. Turcic et al. (2015) identify the risks of supply discontinuity as driving the use of hedging, and with a hedging behavior of each firm heavily depending on that of the other. Either both firms hedge their direct commodity purchases, or neither firm hedges. Our work substantially departs from this study, as our focus is on understanding pass-through contract practices. We introduce financial frictions within our bilateral supply chain, thus necessitating costly financing, and compare the pass-through contract to revenue sharing – a frequently used benchmark. Not only we identify differences in hedging behavior under different supply chain contracts, with or without financial fric-
tions, but also we clearly explain the advantages of pass-through contracts and the environments that are best applied.

Our contributions to the supply chain commodity risk management literature is three fold:

1. We explain the logic behind pass-through contracting practices: the contract not only shifts the risks to downstream buyers with available working capital, but hedging behavior of such buyers reduces the financing costs of all firms in the supply chain; the pass-through contract is an effective way to finance limited working capital suppliers in a stochastic input cost setting.

2. For a frictionless world, we clearly demonstrate the superiority of revenue and cost sharing contracts even in the presence of stochastic input costs, and without the need for any hedging. However, introducing financial frictions breaks down the logic of such contracts, and the created inefficiencies include both double-marginalization and substantial financing costs. For environments in which the financing costs dominate double-marginalization inefficiencies, we observe the pass-through contract dominating the revenue sharing one.

3. The strategic interaction of firms within a supply chain setting, and the risk of discontinuity (from either upstream or downstream) drive firm hedging behavior that depends on the supply chain contract used and the other firms actions. The pass-through contract may require the downstream buyer to hedge all commodity inputs on behalf of the chain. The revenue sharing contract in the presence of financial frictions requires both firms to hedge all commodity inputs in the chain, and with a simultaneity of actions between firms. That is, for hedging to “work,” both firms must hedge.

3 Model Description

Consider the problem of a downstream final goods assembler/manufacturer, \( a \), who faces an inventory problem in that at time \( t_0 \) she must choose a production quantity, \( q \), before she observes a single realization of demand for her final product. The inverse demand curve for the assembler’s output is given by:

\[
p = \xi - D \geq 0,
\]

where \( D \) denotes demand at price \( p \) and \( \xi \geq 0 \) is market potential, which we assume to be a continuous random variable. The assembler sets price \( p \) at time \( t_3 \) when a value \( \xi \) is drawn from a distribution \( F \) (where convenient, \( f \) denotes the corresponding density) with the proviso that she cannot sell more
than she produced for.

![Diagram: Commodity 1 -> Supplier's Component -> Assembler's Final Product -> Commodity 2]

Figure 1: Inputs Required to Produce One Unit of Assembler's Final Product.

To assemble \( q \) units of the final product, the assembler requires \( q \) units of commodity 2 as well as \( q \) components (subassemblies), which are produced by an upstream supplier, \( s \). To produce \( q \) components, the supplier requires \( q \) units of commodity 1. (Figure 1 summarizes the production inputs required to assemble 1 unit of the final product.) For simplicity, unmet demand is lost, unsold stock is worthless, and per unit production and holding costs are zero.

In terms of notation, “\( \pi \)” and “\( \Pi \)” respectively are used to denote the assembler’s and the supplier’s (expected) payoffs. Both the assembler and the supplier are risk-neutral, protected by limited liability, and each firm (company) has \( y_c < \infty, c \in \{a, s\} \), dollars in operating capital.

Due to operational constraints, the sub-assembly production must be completed at time \( t_1 > t_0 \). The final assembly must be completed at time \( t_2 \geq t_1 \) and the assembler’s sales revenue is realized at time \( t_3 \geq t_2 \).

Commodity 1 per-unit price, \( s_1 \), is revealed at time \( t_1 \). Viewed from \( t_0 \), \( s_1 \) is a random variable that takes on one of the values \( s_1^h \) and \( s_1^l \) with probabilities \( \rho_1 \) and \( (1 - \rho_1) \). The commodity 2 per-unit price, \( v_i, i = 1, 2 \) is also a random variable. At time \( t_1 \), \( v_1 \) takes on one of the values \( v_1^h \) and \( v_1^l \) with probabilities \( \rho_2 \) and \( (1 - \rho_2) \). At time \( t_2 \), \( v_2 \) takes on one of the values \( v_2^h, v_2^m \) and \( v_2^l \) with probabilities \( \rho_2^2, 2\rho_2(1 - \rho_2), \) and \( (1 - \rho_2)^2 \). Figure 2, summarizes prices and probabilities graphically.

The discount rate between times \( t_0 \) and \( t_3 \) is taken to be zero. (From CAPM, this is equivalent to assuming that the risk-free rate is zero, that both input commodities are zero beta assets for which there is an empirical support in Pindyck, 1994, and that the market potential risk is diversifiable.)

We consider two supply contract types. The first is a pass-through contract (denoted with subscript “\( pt \)” under which the supplier charges the assembler 100% of his commodity input cost plus fixed markup, \( \gamma \), for each unit produced. The assembler keeps 100% of sales revenue. As such, with the pass-through contract, the downstream assembler faces a unit production cost of \( \omega = s_1 + v_2 + \gamma \) that takes on values \( \omega_{ij} = s_1^i + v_2^j + \gamma, i \in \{l, h\} \) and \( j \in \{l, m, h\} \) with respective probabilities:

\[
\begin{align*}
\mathbb{P}\{\omega = \omega_{hh}\} &= \rho_1\rho_2^2, \\
\mathbb{P}\{\omega = \omega_{hm}\} &= 2\rho_1\rho_2(1 - \rho_2), \\
\mathbb{P}\{\omega = \omega_{hl}\} &= \rho_1(1 - \rho_2)^2, \\
\mathbb{P}\{\omega = \omega_{lh}\} &= (1 - \rho_1)\rho_2^2.
\end{align*}
\]
\[ P\{ \omega = \omega_{lm} \} = 2 \rho_2 (1 - \rho_1)(1 - \rho_2), \quad P\{ \omega = \omega_{ll} \} = (1 - \rho_1)(1 - \rho_2)^2. \]

Without loss of generality, throughout the paper we assume \( \omega_{ll} \leq \omega_{lm} \leq \omega_{lh} \leq \omega_{hl} \leq \omega_{hm} \leq \omega_{hh} \).

The second contract type we consider is revenue sharing (denoted with subscript “rs”), which is a contractual arrangement under which the supply chain members share all revenues as well as commodity input costs. With the revenue sharing contract, \( \gamma = 0 \) and firms share the cost \( \omega \) that we define above.

It is rather important to emphasize that either the supplier or the assembler may have an incentive to default on either supply contract. This could happen if the time \( t_i \) total production cost turns out to be too high in relation to the contractually pre-determined revenues. For this reason, we need to assume that contracts can be legally enforced – but only up to a point. Because firms have limited liability, then no firm can achieve negative wealth.

**Assumption 1.** All supply contracts are enforceable. As such, if a firm \( c \in \{a, s\} \) chooses to terminate a supply contract at time \( t_i \), \( i = 1, 2 \), then it is under the obligation to pay the other party a penalty payment \( P_c \geq 0 \) in the amount of the other party’s expected payoff at time \( t_i \). However, bankruptcy or insolvency of one contracting party means that the other party can costlessly terminate the contract. Finally, all bankrupt or insolvent parties are protected by limited liability.

Firms may have an incentive to hedge their commodity input costs in order to avoid the possibility of defaulting on the supply contract. For this reason we suppose that there exists a futures market at time \( t_0 \). As is convention, the futures prices in this market are set so that the value of each futures contract at inception is zero. The payoff to a futures contract is realized at time \( t_i \), where the payoff is the difference between the futures price and the time \( t_i \) spot price. We use the variable \( n_{j,i}^c, c \in \{a, s\} \) to represent firm \( c \)’s position in the futures market: \( n_{j,i}^c > 0 \) indicates that firm \( c \) has a time \( t_i \) long

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**Figure 2:** Probability Tree for the Commodity Input Costs
position’ of $n_{j,i}^c$ futures contracts in commodity $j = 1, 2$. If the firm $c \in \{a, s\}$ buys – or is long in – $n_{1,i}^c$ futures contracts, its time $t_i$ payoff is $n_{1,i}^c (s_i - S_i)$, where $S_i$ is the futures price of commodity 1. Similarly, if the firm $c \in \{a, s\}$ buys $n_{2,i}^c$ futures contracts, its time $t_i$ payoff is $n_{2,i}^c (v_i - V_i)$, where $V_i$ is the futures price of commodity 2. To avoid pathological cases, the futures prices $S_i, V_i, i = 1, 2$ are low enough so that by hedging, neither firm effectively locks in a negative (expected) profit.

3.1 Timing of Events

Let $i \in \{l, h\}$ and $j \in \{l, m, h\}$. If the contract is pass-through, then define the payments:

$$P_{s,1} = -q s_1^i, \quad P_{s,2} = q (s_1^i + \gamma), \quad P_{s,3} = 0, \quad P_{a,1} = 0, \quad P_{a,2} = -q \omega_{ij}, \quad P_{a,3} = R(q).$$

If the contract is revenue sharing, then define the payments:

$$P_{s,1} = -\lambda_a q s_1^i, \quad P_{s,2} = -\lambda_a q v_2^i, \quad P_{s,3} = \lambda_a R(q), \quad P_{a,1} = -\lambda_a q s_1^i, \quad P_{a,2} = -\lambda_a q v_2^i, \quad P_{a,3} = \lambda_a R(q).$$

With these definitions, the supply chain members’ payoffs under each contract are resolved according to the sequence.

At time $t_0$:
- Firms choose to take positions $n_{j,i}^c \geq 0, c \in \{a, s\}, i, j = 1, 2$ in the futures contracts.
- Assembler, $a$, sets a production quantity, $q$.

This ends time $t_0$. Before time $t_1$ begins the future spot price of commodity 1 price is revealed. At time $t_1$:
- The supplier decides whether or not to produce $q$ components for the assembler; if the supplier decides to produce then he incurs a payment of $P_{s,1}$; the assembler incurs a payment of $P_{a,1}$. If the supplier chooses not to produce, then he faces a penalty $P_{s} \geq 0$.

This ends time $t_1$. Before time $t_2$ begins, the future spot price of commodity 2 and the supplier’s production decision at time $t_1$ are revealed. If the supplier produced at time $t_1$, then at time $t_2$:
- The assembler decides whether or not to produce $q$ units of the final product; if the assembler decides to produce, then she incurs a payment of $P_{a,2}$; the supplier incurs a payment of $P_{s,2}$. If the assembler chooses not to produce, then she faces a penalty $P_{a} \geq 0$.

This ends time $t_2$. Before time $t_3$ begins, market potential $\xi$ is revealed. If the assembler produced at time $t_2$, then, at time $t_3$:
- The assembler sets a retail price, $p$, and the sales revenue, $R(q)$, is realized; the assembler earns $P_{a,3}$ and the supplier receives $P_{s,3}$.

In terms of the information structure, we assume that all market participants can observe the actions taken by all players and can observe all market outcomes, i.e., information is complete. This
assumption can best be justified with emerging empirical research in finance, which reveals that companies commonly incorporate covenant restrictions into supply contracts (e.g., see Smith and Stulz, 1985; Roberts and Sufi, 2009) that help them reduce or eliminate asymmetric information and moral hazard.

The equilibrium concept that will be used is that of a subgame perfect Nash equilibrium (SPNE), which requires that candidate equilibrium strategies are an equilibrium in each and every subgame. In the description of the timing of the pass-through game above, each bullet point represents a subgame. The SPNE is derived by backward induction.

4 Preliminaries: Assembler’s Pricing Decision

Before describing the formal analysis leading to the equilibrium, this section clarifies how the assembler sets a retail price of the final product. Throughout the paper we assume that at time $t_2$ the assembler makes her production decision and then brings what she produced to the market. As such, her most preferred retail price will maximize her sales revenue with the proviso that the total quantity she sells cannot exceed the quantity, $q$, that she produced for at time $t_2$:

$$\max_p p (\xi - p) \quad \text{s.t.} \quad \xi - p \leq q.$$  \hspace{1cm} (1)

To achieve maximum in (1), the assembler will set $p^*(\xi) = \frac{\xi}{2}$ if $\xi \leq 2q$. At this price, she will sell $\frac{\xi}{2}$ units. If, on the other hand, $\xi > 2q$, she will set a price of $p^*(\xi) = \xi - q$, which is the highest price that clears her production quantity, $q$. With these prices, the assembler’s realized sales revenue will be:

$$R(q) = \begin{cases} \frac{\xi^2}{4} & \text{if } \xi \leq 2q; \\ q (\xi - q) & \text{if } \xi > 2q. \end{cases}$$  \hspace{1cm} (2a)

Her expected sales revenue will be:

$$S(q) = \frac{1}{4} \int_0^{2q} \xi^2 f(\xi) d\xi + \int_{2q}^{\infty} q(\xi - q) f(\xi) d\xi,$$  \hspace{1cm} (2b)

with $S'(q) = \int_{2q}^{\infty} (\xi - 2q) f(\xi) d\xi > 0$ and $S''(q) = -2\hat{F}(2q) < 0$. 


5 Contracting Without Borrowing

In this section, we introduce two models: A model of a pass-through contract and a model of a revenue sharing contract, both without borrowing. To allow budget-constrained firms to produce without having to borrow, we take $t_0 < t_1 = t_2 = t_3$. (Note that we continue to assume the exact sequence in the decision process described in Section 3.1.)

With $t_0 < t_1$ both firms remain exposed to input cost volatility while $t_1 = t_2 = t_3$ means each firm incurs costs and revenues simultaneously. As such production is feasible without operating capital, $y_c$, $c \in \{a, s\}$ and without borrowing. We refer to this setup as our benchmark model.

Remark 1. Since $t_1 = t_2$, then $\omega = s_1 + v_2 + \gamma$ takes on values $\omega_{hh}$, $\omega_{hl}$, $\omega_{lh}$, and $\omega_{ll}$ with respective probabilities of $\mathbb{P}\{\omega = \omega_{hh}\} = \rho_1 \rho_2$, $\mathbb{P}\{\omega = \omega_{hl}\} = \rho_1 (1 - \rho_2)$, $\mathbb{P}\{\omega = \omega_{lh}\} = (1 - \rho_1) \rho_2$, and $\mathbb{P}\{\omega = \omega_{ll}\} = (1 - \rho_1)(1 - \rho_2)$.

5.1 A Benchmark Pass-Through Contract

With the pass-through contract, the supplier charges the assembler $(s^i_1 + \gamma)$, $i \in \{l, h\}$ per unit purchased, where $s^i_1$ is the supplier’s raw material cost and $\gamma$ is his fixed processing cost or markup. If the downstream assembler outputs $q$ units of her final product at time $t_2$, then her time $t_3$ revenue will be $R(q)$, which is given by (2a). The sequence of events for the pass-through contract is given in Section 3.1.

We will proceed with the analysis by first assuming that neither firm hedges its input costs and show that in this case, the upstream supplier is subjected to default risk since he agreed to sell a pre-determined output quantity to the downstream assembler at prices that are fixed before all factors affecting the assembler’s productivity are known. Figure 3 summarizes the backward induction steps for this case. We then argue that default can be completely avoided if the assembler enters into $n^a_{1,1} = n^a_{2,2} = q$ futures contracts, i.e., by hedging. Whether or not avoiding default is valuable is determined by comparing the expected payoffs across both cases.

Outcomes Without Hedging. Suppose $n^c_{j,i} = 0$ for all $i, j \in \{1, 2\}$ and $c \in \{a, s\}$. Given a production quantity, $q$, and a market potential, $\xi$, at time $t_3$, the assembler sets retail prices according to the results of Section 4. At time $t_2$, the assembler first gets to observe the supplier’s production decision.
and then commits to production whenever the supplier produced at time \( t_1 \) and \( R(q) - q \omega_{ij} \geq 0 \).\(^2\)

Notice that at time \( t_2 \), the assembler can costlessly default on the pass-through contract because she has no operating capital and is protected by limited liability – see Assumption 1 in Section 3.

Correctly anticipating that he cannot enforce the contract when \( R(q) - q \omega_{ij} \leq 0 \), at time \( t_1 \), the upstream supplier produces only when \( R(q) - q \omega_{ij} \geq 0 \). Then, as a function of \( q \), the supplier’s and the assembler’s expected payoffs at time \( t_0 \) will be:

\[
\Pi_{\text{no hedge}}^{pt}(q) = q \gamma \cdot \mathbb{P} \left\{ \omega \leq \frac{R(q)}{q} \right\} \quad \text{and} \quad \pi_{\text{no hedge}}^{pt}(q) = \mathbb{E} (R(q) - q \omega)^+ . \tag{3}
\]

At time \( t_0 \), the assembler’s most preferred production quantity will be:

\[
d_{\text{no hedge}}^{pt*} = \arg \max_{q \geq 0} \pi_{\text{no hedge}}^{pt}(q).
\]

**Outcomes With Hedging.** The intuition that underlies hedging with the pass-through is that by purchasing \( n_{1,1}^a = n_{2,2}^a = q \) futures contracts\(^3\), the downstream assembler will be able to guarantee the payment of \( \gamma q \) to the upstream supplier. This is because the assembler’s futures contract positions will “pay off” when commodity input costs are high, which is precisely when the assembler defaults on the pass-through if she does not hedge. Since \( n_{1,1}^a = n_{2,2}^a = q \), wipes out the noise in the supplier’s payoff, then his forward market position is irrelevant (i.e., the supplier does not need to hedge).

As before, the analysis proceeds by backward induction. Given a production quantity, \( q \), and a market potential, \( \xi \), at time \( t_3 \), the assembler sets retail prices according to the results of Section 4.

At time \( t_2 \), the assembler first gets to observe the supplier’s production decision and then commits to

\(^2\)By writing, \( R(q) - q \omega_{ij} \geq 0 \), we model the assembler as producing \( q \) units of final product. Alternatively, one could model the assembler as paying the supplier \( q \left( s_1^i + \gamma \right) \) for the \( q \) components and then producing any amount of output less than or equal to \( q \). In such case, the assembler’s cost would be \( q \left( s_1^i + \gamma \right) \); her retail price would be \( p^* = \frac{\xi + v^2_j}{2} \) if \( \xi \leq (2q + v^2_j) \); if \( \xi > (2q + v^2_j) \), the assembler would charge \( p^* = \xi - q \). Finally, her quantity sold would be \( (\xi - p^*) \). Note, however, that modeling the assembler this way does not qualitatively alter any of our results, i.e., the reasons for why firms may prefer pass-through over revenue sharing and vice versa and reasons for why firm(s) may want to hedge.

\(^3\)Note that \( n_{1,1}^a = n_{2,2}^a = q \) means that the assembler fully hedges both commodities. For definition of \( n_{1,1}^a \) and \( n_{2,2}^a \), see Section 3.
production whenever the supplier produced at time $t_1$ and $q v_2^j \leq R(q)$, $i, j \in \{l, h\}$. If the supplier produced at time $t_1$ and $q v_2^j \geq R(q)$, $i, j \in \{l, h\}$, then the assembler could produce as well. However, instead of producing, her preferred strategy is to sell her futures contract positions in commodity 2 and pay the supplier a default penalty of $P_a = q (s_1^j + \gamma)$. In both cases, the supplier will be paid $q (s_1^j + \gamma)$, $i \in \{l, h\}$.

Seeing that he is guaranteed to be paid at time $t_2$, at the time $t_1$, the supplier will produce for sure. (The effect that hedging creates is that it makes it feasible for the assembler to produce in all states of the world $i \in \{l, h\}$. As such, the supplier can enforce the contract in all states of the world $i \in \{l, h\}$.)

Then as a function of the production quantity, $q$, the supplier’s and the assembler’s expected payoffs at time $t_0$ will be:

$$
\Pi_{\text{hedge}}^p(q) = q \gamma \quad \text{and} \quad \pi_{\text{hedge}}^p(q) = \mathbb{E} \left( n_{1,1}^q (s_1 - S_1) + n_{2,2}^q (v_2 - V_2) \right) + \mathbb{E} (R(q) - q v_2)^+ - q \mathbb{E} (s_1 + \gamma) = \mathbb{E} \max \{R(q), q v_2\} - q (S_1 + V_2 + \gamma), \quad (4a)
$$

where $\mathbb{E} \left( n_{1,1}^q (s_1 - S_1) + n_{2,2}^q (v_2 - V_2) \right)$ is the assembler’s payoff from her futures contract position. At time $t_0$, the assembler’s most preferred production quantity will be $q_{\text{hedge}}^{p*} = \arg \max_{q \geq 0} \pi_{\text{hedge}}^p(q)$.

**An Equilibrium Outcome.** Depending on the model parameters, it is possible to have equilibria where (a) neither firm hedges, and (b) the assembler hedges to guarantee the payment to the supplier.

To see how (a) can be supported as an equilibrium, observe that for an arbitrary production quantity, $q$, the assembler’s expected payoff, $\pi_{\text{no hedge}}^p(q)$, given in (3), resembles a payoff of a European call spread option (for further details, see p. 59 in Haug, 1998). Because of this optionality, the assembler’s time $t_0$ expected payoff actually increases in input cost volatility and, consequently, she prefers not to hedge.

However, without hedging the upstream supplier is exposed to default, since his payoff will be $q \gamma$ if $\omega \leq R(q)/q$ and zero otherwise. In contrast, with hedging, the supplier’s payoff will be $\gamma q$ for all $\omega$. The assembler would like to guarantee the supplier’s payoff if the assembler’s expected profit associated with guaranteeing is greater than the expected profit associated with default in some states.

To construct such a case, consider a situation in which the upstream supplier faces a capacity constraint and requires some minimal payoff level, $\Pi^0$, from the capacity that he has available. An important example of this phenomenon is a problem of a contract supplier who is allocating manufac-
turing capacity among multiple buyers.

**Result 1.** Let $\hat{q}$ denote the supplier’s output capacity. If (i) $\hat{q} \leq \max \{ q^\text{hedge}^*, q^\text{no hedge}^* \}$, and (ii) $\hat{q} \gamma \cdot P \{ \omega \leq \frac{R(\hat{q})}{\hat{q}} \} \leq \Pi^0 \leq \hat{q} \gamma$, then the assembler’s expected payoff increases if she hedges.

To understand the above result, observe that condition (i) implies the capacity constraint is binding so that, at time $t_0$, the profit-maximizing assembler will choose a production quantity $\hat{q}$ whether or not she hedges. Condition (ii) implies that without hedging, the supplier will not allocate any capacity to the assembler (i.e., the supplier will not enter into the contract because the contract does not meet her minimum payoff requirement). As such, without hedging, the assembler earns zero and with hedging the assembler’s payoff will be: $\pi^\text{hedge}_t(\hat{q}) = E \max \{ R(\hat{q}), \hat{q} v_2 \} - \hat{q} (S_1 + V_2 + \gamma) \geq 0$. Therefore the assembler’s expected payoff will be greater if she hedges.

**Discussion.** By allowing the supplier to pass his commodity input cost onto the downstream assembler, the pass-through contract is meant to protect the upstream supplier from his commodity input cost risk (e.g., see Zsidisin and Hartley, 2012). However, as a protection from price volatility for the supplier, the pass-through contract disappoints as it assigns a fixed payoff to the supplier (which he does not receive for sure) and a spread option payoff to the assembler. While the ability to exercise her spread option clearly benefits the assembler, this benefit is not necessarily shared with the supplier who simply prefers to receive her fixed payment.

### 5.2 A Benchmark Revenue Sharing Contract

With the revenue sharing contract, at time $t_1$, the upstream supplier charges the downstream assembler $(1 - \lambda)q \ s_1^i$, $\lambda \in (0, 1)$, $i \in \{l, h\}$ per unit of his raw material purchases. The assembler charges the supplier $\lambda q \ v_2^j$, $j \in \{l, h\}$ per unit of her raw material purchases _plus_ she gives the supplier $\lambda$ share of sales revenue $R(q)$. (Note that, in this paper, it is not our goal to determine $\lambda$. Instead, we simply report the range of $\lambda$ that our model can support in equilibrium – for a discussion on how $\lambda$ could be set, see Cachon, 2003, p.246.) For notational convenience, we let $\lambda_s = \lambda$ and $\lambda_a = (1 - \lambda)$. The sequence of events for the revenue sharing contract is given in Section 3.1.
Revenue Sharing SPNE. The same steps that led to the equilibrium pass-through contract of Section 5.1 now reveal that the supplier’s and the assembler’s expected payoffs at time $t_0$ will be:

$$\Pi_{rs}^{no \text{ hedge}}(q) = \lambda_s \mathbb{E}(R(q) - s_1 q - v_2 q)^+ \quad \text{and} \quad \pi_{rs}^{no \text{ hedge}}(q) = \lambda_a \mathbb{E}(R(q) - s_1 q - v_2 q)^+.$$  

With the revenue sharing contract, the assembler’s most preferred production quantity $q_{rs}^{no \text{ hedge}}^{*}$ maximizes the expected profit of the integrated supply chain, i.e., $q_{rs}^{no \text{ hedge}}^{*} = \arg \max_{q \geq 0} \mathbb{E}(R(q) - s_1 q - v_2 q)^+$. Therefore the revenue sharing contract with penalties and without hedging achieves first-best.

Finally, neither supply chain member will hedge in equilibrium. This follows because each supply chain member’s expected profit given in (5) is again a payoff of a European call spread option (see Haug, 1998, p. 59). Because of this optionality, each firms’ time $t_0$ expected payoff will actually increase in input cost volatility.

Discussion. The revenue sharing contract has been widely recognized for its ability to coordinate a decentralized supply chain in a setting where demand is stochastic and both production costs and retail price are fixed (Cachon, 2003). Above we show that the contract can also be a highly effective risk management tool. In particular, the revenue sharing contract with penalties continues to achieve first-best and can arbitrarily allocate the integrated system payoff between the supply chain members when all members face stochastic input cost, stochastic demand, and when the retail price is endogenous. Moreover, the contract achieves this outcome without the need to hedge, which is appealing because not all supply chain managers necessarily have the expertise to trade in the financial markets.

5.3 Comparison of Pass-Through and Revenue Sharing Contracts Without Borrowing

It is straightforward to confirm that the pass-through contract of Section 5.1 achieves first-best only when $\gamma = 0$, at which point the supplier’s expected payoff is zero. This observation leads to our preliminary result that deals with firms’ preferences over the benchmark contracts.

Result 2. The benchmark revenue sharing contract pareto dominates the benchmark pass-through contract.
6 Contracting with Borrowing

Consider now the case in which the supplier needs to borrow $L_s = (s^i q - y_s)^+$ at time $t_1$ and the assembler needs to borrow $L_a = [q \omega_{ij} - y_a]^+$ at time $t_2$. To create the need for borrowing, we increase lead times (i.e., we take $t_0 < t_1 < t_2 < t_3$), so that neither firm is able to use its operating revenues to cover its operating costs.

To model debt, we rely on standard modeling assumptions. All lenders are risk-neutral. All borrowers are protected by limited liability, and so their net worth cannot take negative values. All lenders behave competitively in the sense that the loan, if any, makes zero profit (see Tirole, 2006, §3.2.1). In case of default, the lenders incur a proportional bankruptcy cost $0 \leq \alpha \leq 1$. (For an additional discussion on bankruptcy costs, see Kouvelis and Zhao, 2016, .) Together with assumptions from Section 3, we now re-examine the benchmark contracts of Section 5.

6.1 Pass-Through Contract

With borrowing, the pass-through contract payoffs are resolved according to the same sequence as that of the pass-through contract of Section 5.1 with the proviso that both firms must now borrow. Borrowing adds a domain where the supply chain breaks down because one of the firms is credit-rationed (i.e., denied a loan), which occurs when the lenders are not guaranteed a zero profit in expectation.

As in Section 5, the SPNE is derived by backward induction. Figure 3 applies here as well with the added consideration that at times $t_1$ and $t_2$, the bank decides whether or not to lend to the supplier and to the assembler and sets the borrowing cost.

To streamline our presentation, we present all our subgame equilibria for the case when firms do not hedge (i.e., we assume $n^c_{j,i} = 0$ for all $i, j$, and $c$). Then in our description of the SPNE, we explain how the results in each subgame change if each firm adopts the equilibrium hedging policy and determine when firms would hedge.

6.1.1 Preliminaries: Loan Contract Analyses

Since availability of financing will be a necessary prerequisite for production, then the subgame analysis must be preceded by the analysis of the firm’s loan contracts.
Assembler’s Loan Contract. In order to produce at time $t_2$, the downstream assembler needs to borrow $L_a = [\omega_{ij} q - y_a]^+$. A decision to produce will generate a sales revenue of $R(q) \geq 0$ at time $t_3$, given by (2a). Since $R(q, \xi = 0) = 0$ and $\partial R(q)/\partial \xi > 0$, then it follows that the assembler’s revenue will be sufficient to repay $L_a$ plus interest only if the market potential, $\xi$, is sufficiently high; otherwise the assembler will default.

Formally, we define $2k_a$ to be the assembler’s default threshold such that if the realized market size, $\xi < 2k_a$, the assembler will default on the loan, $L_a$, due to insufficient demand; for values $\xi > 2k_a$, the loan, $L_a$, will be repaid as agreed. Then $\frac{1}{4} (2k_a)^2 = k_a^2 = L_a(1 + r_a)$, where $r_a$ is the rate of interest on the assembler’s loan, $L_a$.

There are two cases to consider: $k_a \leq q$ and $k_a > q$. If $k_a \leq q$, then $\xi = 2k_a$ means that the assembler’s realized revenue at the default threshold is $\frac{1}{4} (2k_a)^2 = k_a^2$ and the lender’s expected payoff from lending $L_a$ is:

$$S_b(k_a, q) = \frac{1}{4} \int_0^{2k_a} (1 - \alpha)\xi^2 f(\xi) d\xi + L_a(1 + r_a) \int_{2k_a}^{\infty} f(\xi) d\xi$$  \hspace{1cm} (6)

If $k_a > q$, then $\xi = 2k_a$ means that the assembler’s realized revenue at the default threshold is $q(\xi - q) = q(2k_a - q)$. Surprisingly, this is a case when the assembler can default even if she clears her entire inventory, $q$. To see how high market potential helps the assembler avoid default, consider that she clears her inventory at an optimal retail price of $(\xi - q)$ to earn a revenue of $q(\xi - q)$ (see Section 4). Therefore strictly higher (lower) $\xi$, yields a strictly higher (lower) retail price. Because the number of units sold remains unchanged, her sales revenue also strictly increases (decreases). It follows that if $k_a > q$, then by lending $L_a$ to the assembler, the bank earns

$$S_b(k_a, q) = \frac{1}{4} \int_0^{2q} (1 - \alpha)\xi^2 f(\xi) d\xi + \int_{2q}^{2k_a} (1 - \alpha)q(\xi - q) f(\xi) d\xi + L_a(1 + r_a) \bar{F}(2k_a)$$

on expectation.

The following Lemma 1 introduces sufficient conditions under which the assembler can arrange financing at time $t_2$.

**Lemma 1.** Suppose the distribution of the market potential, $\xi$, is IFR. Then there exists a production quantity $\bar{q}(\omega_{ij})$ such that, at time $t_2$, the assembler can borrow to produce up to $\bar{q}(\omega_{ij})$ units. Moreover, $\bar{q}(\omega_{ij})$ decreases in $\omega_{ij}$. For $q \leq \bar{q}(\omega_{ij})$, we have both $k_a(q)$ and $(2k_a(q) - q)$ increasing in $q$.

Finally, in cases when the assembler can borrow, her equilibrium cost of borrowing must satisfy a
familiar break-even condition (e.g., see Tirole, 2006, §3.2.1):

\[ S_b(k_a, q) = [\omega_{ij} q - y_a]^+ = L_a(q). \] (7)

**Supplier’s Loan Contract.** In order to produce at time \( t_1 \), the upstream supplier needs to borrow \( L_s = (s_i^1 q - y_s)^+ \), \( i \in \{l, h\} \) and his ability to repay \( L_s \) will depend on the downstream assembler’s decision to produce at time \( t_2 \): If the assembler decides to produce, the supplier’s bank receives \( \min\{(s_i^1 q - y_s)^+(1 + r_s), (s_i^1 + \gamma)q\} \), where \( r_s \) is the rate of interest on the supplier’s loan. However, if the assembler does not produce, the supplier’s bank receives only \( \min\{(s_i^1 q - y_s)^+(1 + r_s), P_a, y_a\} \).

Since the rate of interest, \( r_s \), paid to the lenders must again guarantee zero profit in expectation, then it must be true that

\[ (s_i^1 q - y_s)^+ = \rho_2 \min\{(s_i^1 q - y_s)^+(1 + r_s), P_a, y_a\} + (1 - \rho_2) \min\{(s_i^1 q - y_s)^+(1 + r_s), (s_i^1 + \gamma)q\}, \]

where \((1 - \rho_2)\) is the probability the assembler produces at time \( t_2 \).

Now, assume \( L_s = s_i^1 q - y_s > 0 \) and consider that the case when \( \min\{s_i^1 q - y_s, y_a, P_a\} = s_i^1 q - y_s \).

Then

\[ \rho_2 \min\{s_i^1 q - y_s, P_a, y_a\} + (1 - \rho_2) \min\{s_i^1 q - y_s, (s_i^1 + \gamma)q\} = \rho_2(s_i^1 q - y_s) + (1 - \rho_2)(s_i^1 q - y_s) = s_i^1 q - y_s, \]

implying that \( \min\{y_a, P_a\} \geq (s_i^1 q - y_s) \) is sufficient for the supplier to have guaranteed access to an interest-free loan.

On the other hand, if \( \min\{s_i^1 q - y_s, y_a, P_a\} = y_a \), then \( r_s > 0 \), since \( L_s \) will not be paid in full if assembler chooses not to produce at time \( t_2 \). Specifically, we have

\[ s_i^1 q - y_s = \rho_2 y_a + (1 - \rho_2) \min\{(s_i^1 q - y_s)(1 + r_s), q(s_i^1 + \gamma)\}. \] (8)

Consequently,

\[ r_s = \frac{s_i^1 q - y_s - \rho_2 y_a}{(1 - \rho_2)(s_i^1 q - y_s)} - 1 = \frac{\rho_2}{1 - \rho_2} \left[ 1 - \frac{y_a}{s_i^1 q - y_s} \right]. \]

Then, we need \((s_i^1 + \gamma)q \geq (s_i^1 q - y_s)(1 + r_s)\), which yields \( y_a \geq \left( s_i^1 - \frac{1 - \rho_2 \gamma}{\rho_2} \right) q - \frac{y_s}{\rho_2} \) otherwise, the supplier’s bank will make less than zero profit in expectation.

**Lemma 2.** The sufficient conditions under which the upstream supplier can arrange for financing at
time $t_1$ is

$$y_a \geq \left( s^i_1 - \frac{1 - \rho_2}{\rho_2} \gamma \right) q - \frac{y_a}{\rho_2}.$$  \hfill (9)

### 6.1.2 Assembler’s Pricing Decision

Given a time $t_2$ production quantity, $q$, the assembler’s optimal retail price maximizes her sales revenue with the proviso that she cannot sell more than $q$. The assembler’s pricing decision at time $t_3$ gives rise to expected sales revenue, $R(q)$, given by Equation (2a). For further details, see Section 4.

### 6.1.3 Assembler’s Decision to Produce

At time $t_2$, the assembler gets to observe the state of the world, $\omega_{ij}$, $i \in \{l,h\}$, $j \in \{l,m,h\}$, and decides whether or not she wants to produce. Let $x_{ij}$ be an indicator variable for the event that the assembler produces in state $(i,j)$, $i \in \{l,h\}$, $j \in \{l,m,h\}$. That is,

$$x_{ij} = \begin{cases} 
1 & \text{if the assembler produces when she faces input cost } \omega_{ij}, \ i \in \{l,h\}, \ j \in \{l,m,h\}, \\
0 & \text{otherwise.} 
\end{cases}$$

Then depending on the value of $x_{ij}$, the assembler’s expected payoffs at time $t_2$ will be:

$$\pi(q, \omega_{ij} \mid x_{ij}) = \begin{cases} 
\pi(q, \omega_{ij} \mid 1) = S(q) - S_t(k_a, q) - q\omega_{ij}, & \text{ (i.e., the assembler produces at time } t_2), \\
\pi(q, \omega_{ij} \mid 0) = -\min\{P_a, y_a\}, & \text{ (i.e., the assembler defaults at time } t_2). 
\end{cases}$$ \hfill (10)

In the upper prong, $S(q)$ is the assembler’s expected sales revenue viewed from time $t_2$ (see Equation 2b). $S_t(k_a, q)$, given by Equation (11), is a sales revenue, which the assembler must surrender in expectation to the lenders due to default on her loan, $L_a$. (Note that default occurs when the market potential for the assembler’s final product, $\xi$, is too low.) Formally,

$$S_t(k_a, q) = \begin{cases} 
\frac{1}{4} \int_0^{2k_a} \xi^2 f(\xi)d\xi + k_a^2 \int_{2k_a}^{\infty} f(\xi)d\xi, & k_a \leq q, \\
\frac{1}{4} \int_0^{2q} \xi^2 f(\xi)d\xi + \int_{2q}^{2k_a} q(\xi - q) f(\xi)d\xi + q(2k_a - q) \int_{2k_a}^{\infty} f(\xi)d\xi, & k_a > q.
\end{cases}$$ \hfill (11)

Finally, $\omega_{ij}$, $i \in \{l,h\}$, $j \in \{l,m,h\}$ is the assembler’s commodity input cost at time $t_2$. The expression in the lower prong of (10) is the penalty the assembler faces is if she chooses to default on the pass-through contract at time $t_2$.

In order for the capital-constrained assembler to produce when facing input cost $\omega_{ij}$, $i \in \{l,h\}$, $j \in \{l,m,h\}$, production must be both feasible and optimal. Optimality dictates that a decision to
produce must yield an expected profit that exceeds the penalty \(\min\{y_a, P_a\}\). That is,
\[
\omega_{ij} \leq \frac{S(q) - S_t(k_a, q) + \min\{y_a, P_a\}}{q}.
\] (12)

However, since the decision to produce involves borrowing, then feasibility (Lemma 1) also requires
\(q \leq \bar{q}(\omega_{ij}), i \in \{l, h\}, j \in \{l, m, h\}\).

We can show that when the market potential, \(\xi\), is IFR, then both the feasibility and the optimality
conditions are monotone in \(\omega_{ij}\). That is to say, as the assembler’s commodity input costs increase,
she will become increasingly likely to default at time \(t_2\) – either because defaulting on the pass-
through contract will be cheaper than producing or because the assembler will be denied a loan. The
monotonicity of the feasibility condition follows from Lemma 1. The monotonicity of the optimality
condition follows from the next Lemma 3.

**Lemma 3.** Let \(\pi(q, \omega_{ij} | x_{ij})\) be given by (10). Then \(\pi(q, \omega_{ij} | x_{ij})\) decreases as \(\omega_{ij}\) increases.

### 6.1.4 Supplier’s Decision to Produce

At time \(t_1\), the supplier gets to observe the values of \((s^j_i, v^j_1), i, j \in \{l, h\}\), knowing that they’ll
determine the values of \(\omega_{ij}, i \in \{l, h\}\) and \(j \in \{l, m, h\}\) that the assembler will face at time \(t_2\) and
thereby the assembler’s decision to produce at time \(t_2\). If \((s_1, v_1) = (s^j_i, v^h_1), i \in \{l, h\}\), then the
assembler will face \(\omega_{ih}\) with probability \(\rho_2\) and \(\omega_{im}\) with probability \((1 - \rho_2)\). If \((s_1, v_1) = (s^j_i, v^l_1), i \in \{l, h\}\), then the assembler will face \(\omega_{im}\) with probability \(\rho_2\) and \(\omega_{il}\) with probability \((1 - \rho_2)\).

If the supplier chooses not to produce at time \(t_1\), then she pays the assembler \(\min\{y_s, P_s\}\). Otherwise,
viewed from time \(t_1\), his expected payoff will be:
\[
\Pi(q, s^i_1 | x_{iy}, x_{iz}) = \rho_2 \Pi_x(q, s^i_1 | x_{iy}) + (1 - \rho_2) \Pi_x(q, s^i_1 | x_{iz}), i \in \{l, h\}, (y, z) \in \{(h, m), (m, l)\},
\] (13)
where
\[
\Pi_x(q, s^i_1 | x) = \begin{cases} 
\gamma q - L_s r_s & \text{if } x = 1; \\
\max\{\min\{y_a, P_a\} - (qs^i_1 + L_s r_s), -y_s\} & \text{if } x = 0.
\end{cases}
\]

Correctly anticipating the assembler’s production decision in each state \((i, j), i \in \{l, h\}, j \in \{l, m, h\}\),
the supplier will produce whenever,
\[
\Pi(q, s^i_1 | x_{iy}, x_{iz}) \geq -\min\{y_s, P_s\}.
\] (14)

Condition (14) will hold automatically if \(x_{iy} = 1\) and \(x_{iz} = 1\) and becomes uninteresting if \(x_{iy} = 0\) and
In former (latter) case, the assembler will produce (default on the pass-through contract) for sure at time $t_2$ and the supplier will (not) be able to borrow at time $t_1$.

If $x_{iy} = 0$ and $x_{iz} = 1$, then differentiation of $\Pi(q, s_1^i | 0, 1)$ reveals that the left side of (14) increases (decreases) in the production quantity, $q$, if $0 \leq \rho_2 \leq (>) \frac{\gamma}{\gamma + s_1^i}$. Therefore when $x_{iy} = 0$ and $x_{iz} = 1$, the supplier’s production decision reduces to a simple decision rule on $q$: If $\rho_2 \leq (>) \frac{\gamma}{\gamma + s_1^i}$, then the supplier will optimally choose to produce whenever $q$ is sufficiently high (low). However, because the supplier must borrow in order to produce, then both Lemma 2 and condition (14) must hold.

### 6.1.5 Pass-Through SPNE

#### Outcomes Without Hedging.

Suppose $n_{ij}^c = 0$, for all $i, j \in \{1, 2\}, c \in \{a, s\}$. Then at time $t_0$, the assembler’s most preferred production quantity will be

$$
q_{p}^{*\text{hedge}} = \arg \max_{q, x} \mathbb{E}(q, \omega | x),
$$

s.t. $$
\Pi(q, s_1^i | x_{iy}, x_{iz}) + \min\{y_s, P_s\} \geq 0, i \in \{l, h\}, (y, z) \in \{(h, m), (m, l)\}
$$

$$
\left(s_1^i - \frac{1 - \rho_2}{\rho_2}\gamma\right) q - \frac{y_s}{\rho_2} \leq y_a, i \in \{l, h\},
$$

$$
q \leq \bar{q}(\omega_{ij}), i \in \{l, h\}, j \in \{l, m, h\}.
$$

By writing $\mathbb{E}(q, \omega | x)$ (15a) we mean that the expectation is taken over $\omega$ conditional on the values of $x = (x_{hh}, x_{hm}, x_{hl}, x_{lh}, x_{lm}, x_{ll})$, which indicate whether or not the assembler plans on producing in state $(i, j)$: If the assembler plans on producing when $\omega = \omega_{ij}$, then $x_{ij} = 1$; otherwise $x_{ij} = 0$. In Appendix, we establish that $\mathbb{E}(q, \omega | x)$ is concave in $q$ (Lemma B.1.)

The constraint (15b) is an incentive compatibility constraint for the supplier, which ensures that the supplier delivers a component to the assembler on all states $(i, j)$ in which the assembler wants to produce. The remaining constraints are incentive compatibility constraints for the lenders. The constraint (15c) ensures that the supplier can borrow. Similarly, the constraint (15d) ensures that the assembler can borrow.

Recall from Lemma 1 that the constraint (15d) implies a monotone behavior: As $\omega_{ij}, i \in \{l, h\}, j \in \{l, m, h\}$ increases, the quantity, $q$, that assembler can process at time $t_2$ decreases because
the assembler will be credit-rationed. Correctly anticipating a break-down in the supply chain, the incentive compatibility constraints (15b) and (15c), may lead the supplier to default at time \( t_1 \). Using this reasoning, it can be shown that, in equilibrium, the un-hedged supply chain will operate in one of eight possible ways, which are summarized graphically in Figure 4.

*It is important to note that if neither firm hedges, then it does not imply that the supply chain will break down.* If (12) holds for \( \omega_{hh} \), then the assembler can always borrow at time \( t_2 \). This means that the supplier will always be able to enforce the pass-through contract at time \( t_2 \). Anticipating this, the supplier will always choose produce at time \( t_1 \). Graphically, this case is shown in Figure 4a.

### Figure 4: Supply Chain Performance Without Hedging

**Note:** \( \varphi_2 \) is a (conditional) probability that the downstream assembler defaults on the pass-through contract at time \( t_2 \) given that the upstream supplier produced at time \( t_1 \). At time \( t_1 \), the supplier produces with probability \( \varphi_1 \).

**Outcomes With Hedging.** The intuition that underlies our model of hedging with the pass-through is that by purchasing \( n_{1,1}^a = n_{2,2}^a = q \) futures contracts, the downstream assembler wipes out the variability in the commodity input costs that she faces at time \( t_2 \). As such, with hedging, the downstream assembler will be able to borrow and produce in all future states of the world. Foreseeing this, the upstream supplier will also produce in all future states of the world, which leads to the following
expected payoffs:

\[ \pi_{\text{hedge}}^p(t)(q) = E \max \{ S(q) - S_t(k_a(\mathbb{E}\omega)), q v_2 \} - q \mathbb{E}\omega \quad \text{and} \quad \Pi_{\text{hedge}}^p(q) = \gamma q. \]  \hspace{1cm} (16)

There payoffs are analogous to the expected payoffs given in Equation (4a) with the proviso that the assembler’s expected payoff in (16) contains the (expected) financing cost, \( S_t(k_a(\mathbb{E}\omega)) \), given by (11).

In contrast, the supplier’s payoff does not contain any financing cost because when the assembler hedges, the supplier can guarantee himself a payoff of \( \gamma q \) at time \( t_2 \), which allows him to borrow at the risk-free rate (zero) at time \( t_1 \).

With hedging, at time \( t_0 \), the assembler’s most preferred production quantity, \( q_{\text{hedge}}^{pt} \), will solve (15), except that instead of the random variable \( \omega \), the assembler will face \( \mathbb{E}\omega \) in all future states of the world.

**Equilibrium.** Analytically, we are able to identify two equilibria in which the assembler hedges by buying \( n_{1,1}^a = n_{2,2}^a = q \) futures contracts at time \( t_0 \): In the first equilibrium, the assembler hedges when the supplier faces a capacity constraint, \( \hat{q} \), and requires some minimal payoff level, \( \Pi^0 \), from the capacity that he has available. Hedging is valuable because it guarantees the supplier the highest payoff that he can achieve, \( \gamma \hat{q} \). (For details, see our Result 1 in Section 5.1).

In the second equilibrium, hedging is beneficial to the assembler because it allows her to reduce her expected financing costs. (As such, this equilibrium does not exist in the benchmark case.) The basic logic can be understood as follows. If the assembler does not hedge, then variability in her commodity input costs leads to variability in the amount that she has to borrow at time \( t_2 \). In the proof of the next Proposition 3, we show that the financing cost is convex, increasing in the amount raised. With convex, increasing costs, hedging is valuable because it allows the assembler to avoid scenarios in which the financing costs are excessively high.

**Result 3.** If (i) Condition (12) holds for \( \omega_{hh} \), and (ii) \( q \leq \bar{q}(\omega_{hh}) \) (cf. Lemma 1), then the assembler will hedge by taking \( n_{1,1}^a = n_{2,2}^a = q \).

In our model, reducing fluctuations in the amount that the assembler needs to borrow at time \( t_2 \) is only valuable when the fluctuations in the commodity input costs are not too high. In cases, when costs fluctuate a lot and production costs exceed operating revenues by a lot, then the assembler does not need to borrow at all. This is because her optimal response to is to shut down production and
default on the pass-through contract. Due to this shut down option, conditions (i) and (ii) in Result 3 are required to ensure that hedging is beneficial to the assembler. (In words, condition (i) and (ii) ensure that the assembler wants to produce – and therefore borrow – in all states of the world.)

6.2 Revenue Sharing Contract

With the revenue sharing contract with borrowing, the firms’ payoffs are resolved according to the same sequence as that of the revenue sharing contract of Section 5.2, except that each firm \( c \in \{a, s\} \) must borrow \( L_{c,1} \) at time \( t_1 \) and \( L_{c,2} \) at time \( t_2 \). (We specify the amounts \( L_{c,1} \) and \( L_{c,2} \) in Section 6.2.1, which we present next.) Since the formal analysis that leads to the SPNE proceeds along the steps of Section 6.1, then to avoid repetition, we explain how the results for the revenue sharing contract can be recovered from the results for the pass-through contract. As in Section 6.1, the model is solved by first examining the loan contracts; then stage 5 characterizes the equilibrium retail price; stage 4 and 3 characterize the firms’ production decisions; stage 2 determines the equilibrium production quantity, and stage 1 determines whether or not the firms want to hedge.

6.2.1 Firm’s Loan Contracts

The revenue sharing contract differs from the pass-through contract in that each firm borrows twice – first at time \( t_1 \) and then again at time \( t_2 \). This is seen graphically in Figure 5.

To see why firms borrow twice, consider that in order for the supplier, \( s \), to produce at time \( t_1 \), each firm \( c \in \{a, s\} \) must borrow \( L_{c,1} = (\lambda_c s^1_i q - y_c)^+ \), \( i \in \{l, h\} \) to pay for its \( \lambda_c \)-share of the

![Figure 5: Loan Agreements under the Revenue Sharing Contract](image-url)
commodity input cost, \( \lambda_c s_1^i q \). The zero-profit constraint for the lenders can be written as

\[
L_{c,1} = \rho_2 y_c + (1 - \rho_2) \min \{ L_{c,1}(1 + r_{c,1}), L_{c,2} \},
\]

where \( L_{c,2} \geq L_{c,1}(1 + r_{c,1}) \), which yields the following to Lemma 2.

**Lemma 4.** A sufficient conditions under which each firm \( c \in \{a, s\} \) can arrange for financing at time \( t_1 \) is

\[
y_c \geq L_{c,2} + \frac{L_{c,1} - L_{c,2}}{\rho_2}, \quad c \in \{a, s\}.
\]  

(17)

Then, in order for the assembler to produce at time \( t_2 \), each firm \( c \in \{a, s\} \) will borrow

\[
L_{c,2} = \left[ \lambda_c (s_1^i q + v_2^j q) - y_c \right]^+, \quad i \in \{l, h\}, \ j \in \{l, m, h\}
\]

and use it to both repay \( L_{c,1} \) and to pay for its \( \lambda_c \)-share of the commodity input cost, \( \lambda_c v_2^j q \). At time \( t_2 \), each firm faces the same situation as the assembler faces under the pass-through contract at time \( t_2 \) with proviso that each firm pays for \( \lambda_c \)-share of the production cost and receives \( \lambda_c \)-share of the sales revenue. As such, the expression for \( S_b(k_c, q) \), \( c \in \{a, s\} \) given in Section 6.1.1 can be re-written as:

\[
S_b(k_c, q) = \frac{1}{4} \int_0^{2q} \lambda_c (1 - \alpha) \xi^2 f(\xi) d\xi + \int_{2q}^{2k_c} \lambda_c (1 - \alpha) q (\xi - q) f(\xi) d\xi + L_{c,2}(1 + r_{c,2}) \bar{F}(2k_c), \quad c \in \{a, s\}.
\]

With this new expression for \( S_b(k_c, q) \), we adapt the proof of Lemma 1 to establish the following result, which is a sufficient condition under which each firm \( c \in \{a, s\} \) can borrow at time \( t_2 \).

**Lemma 5.** Suppose the distribution of the market potential, \( \xi \), is IFR. Then there exists a production quantity \( \bar{q}(\omega_{ij}) \) such that, at time \( t_2 \), each firm \( c \in \{a, s\} \) can borrow to produce up to \( \bar{q}(\omega_{ij}) \) units. Moreover, \( \bar{q}(\omega_{ij}) \) decreases in \( \omega_{ij} \).

Finally, in cases when both firms can borrow, their equilibrium costs of borrowing must again satisfy the break-even condition:

\[
S_b(k_c, q) = L_{c,2}(q), \quad c \in \{a, s\},
\]

(18)

which is analog of Equation (7) will be the zero-profit constraint for the lenders.

### 6.2.2 Assembler’s Retail Pricing Decision and the Firms’ Production Decisions

Taking the production quantity \( q \) as given, at time \( t_3 \), the assembler sets a retail price according the results of Section 4.

To characterize the firms’ production decisions, set \( \omega_{ij} = (s_1^i + v_2^j) \), \( i \in \{l, h\}, \ j \in \{l, m, h\} \). Then
in Section 6.1.3, replace Equation (10) with
\[
\pi(q, \omega_{ij} | x_{ij}) = \begin{cases} 
\pi(q, \omega_{ij} | 1) = \lambda_a [S(q) - S_t(k_a, q) - q \omega_{ij}], & \text{if the assembler produces at time } t_2 \\
\pi(0, q, \omega_{ij} | 0) = -\min\{P_a, y_a\}, & \text{if the assembler defaults at time } t_2 
\end{cases}
\] (19)
and Equation (11) with
\[
S_t(k, q) = \begin{cases} 
\lambda_c \left[ \frac{1}{4} \int_0^{2k} \xi^2 f(\xi) d\xi + k_a \int_{2k}^\infty f(\xi) d\xi \right], & k_a \leq q, \\
\lambda_c \left[ \frac{1}{4} \int_0^{2q} \xi^2 f(\xi) d\xi + \int_{2q}^{2k_a} q(\xi - q) f(\xi) d\xi + q(2k_a - q) \int_{2k_a}^\infty f(\xi) d\xi \right], & k_a > q. 
\end{cases}
\] (20)
The assembler’s feasibility condition (12) (i.e., condition that guarantees that the assembler can borrow at time \(t_2\)) stays the same, except that the “numerical” values of \(k_a\) and \(P_a\) change. That is, to produce, the assembler requires:
\[
\omega_{ij} \leq \frac{S(q) - S_t(k_a, q) + \min\{y_a, P_a\}}{q},
\] (21)
where \(S(q)\) is the assembler’s expected sales revenue viewed from time \(t_2\) (see Equation 2b). In Section 6.1.4, replace Equation (13) with small
\[
\Pi_x(q, s_{i1} | x) = \begin{cases} 
\lambda_a [S(q) - S_t(k_a, q) - q \omega_{ij}] & \text{if } x = 1; \\
\max\{\min\{y_a, P_a\} - (\lambda_a q s_{i1} + L_a r_s), -y_s\} & \text{if } x = 0.
\end{cases}
\] (22)
For the supplier, the feasibility condition is (17), which requires that both firms must have some working capital in order to produce.

6.2.3 Revenue Sharing SPNE

Outcomes Without Hedging. Without hedging, \(n_{j,i}^c = 0\), for all \(i, j \in \{1, 2\}, c \in \{a, s\}\). To derive the assembler’s optimal order quantity without hedging, consider the optimization problem (15), and replace the constraint \(s_{i1}^i - \frac{1 - \rho_2^2}{\rho_2} q - \frac{\mu}{\rho_2} \leq y_a, i \in \{l, h\}\) with (17). For \(\pi(q, \omega_{ij} | x_{ij})\) and \(\Pi_x(q, s_{i1}^i | x)\) use expressions given in (19) and (22).

Outcomes With Hedging. Whereas with the pass-through contract, the upstream supplier passes the commodity price risk onto the downstream assembler who then hedges it, with the revenue sharing contract, each firm \(c \in \{a, s\}\) pays \(\lambda_c q s_{1}^i\) at time \(t_1\) and \(\lambda_c q v_2\) at time \(t_2\). Therefore each firm is exposed to price risk from both commodities 1 and 2. The logic behind this result is the same as
the logic behind Result 3, where the assembler hedges to reduce her expected borrowing costs. (For a discussion, see Section 6.1.5 where we contrast Result 3 with its variant put forth by Froot et al. (1993).)

**Result 4.** If (i) Conditions (17) and (21) simultaneously hold for \( \omega_{hh} \), and (ii) \( q \leq \bar{q}(\omega_{hh}) \) (cf. Lemma 5), Then, each firm \( c \in \{a,s\} \) will hedge by purchasing \( n_{c,1}^\varnothing = n_{c,2}^\varnothing = \lambda_c q, \ c \in \{a,s\} \) futures contracts at time \( t_0 \).

Although the logic behind Results 3 and 4 is the same. There is an important difference between the two results: With the pass-through contract only the downstream assembler hedges – the supplier receives the full benefit of hedging although he does not hedge. With the revenue sharing contract both firms must hedge.

**Corollary 1.** In equilibrium, either both firms \( c \in \{a,s\} \) will hedge with \( n_{c,1}^\varnothing = n_{c,2}^\varnothing = \lambda_c q \) or neither firm hedge, i.e., \( n_{c,1}^\varnothing = n_{c,2}^\varnothing = 0 \).

Corollary 1 is similar in spirit to hedging behavior in a supply chain with a wholesale price contract (see Turcic et al., 2015). To understand the above result, observe that if a firm \( c \in \{a,s\} \) enters into \( n_{c,1}^\varnothing \geq 0 \) and \( n_{c,2}^\varnothing \geq 0 \) \( c \in \{a,s\} \) futures contracts, then at time \( t_1 \) (2), its payoff from the futures contract position will be positive when \( S_1 \leq s_1 \) \( (V_2 \leq v_2) \) and negative when \( S_1 > s_1 \) \( (V_2 > v_2) \). The result is driven by the risk of default on the futures contract, which arises as follows: If firm \( c \in \{a,s\} \) enters into \( n_{c,1}^\varnothing = n_{c,2}^\varnothing = \lambda_c q \) futures contract and its supply chain counterpart, say firm \( b \in \{a,s\}, b \neq c \) does not hedge (i.e., if \( n_{b,1}^\varnothing = n_{b,2}^\varnothing = 0 \), then firm \( c \) is at risk of default on its futures contract position \( n_{c,2}^\varnothing \). The risk of default arises when firm \( b \) defaults on the revenue sharing contract at time \( t_1 \) (because commodity input costs at time \( t_1 \) turn out to be high) and commodity input costs at time \( t_2 \) turn out to be low, exposing firm \( c \) to a negative payoff from the futures contract position. So firm \( c \in \{a,s\} \) will not hedge if she anticipates that her supply chain counterpart will not hedge.

To derive the assembler’s optimal order quantity with hedging, consider the optimization problem (15), and replace the constraint \( \left( s_i^l - \frac{1-\rho_2 \gamma}{\rho_2^2} \right) q - \frac{\nu_i}{\rho_2} \leq y_a, \ i \in \{l,h\} \) with (17). For \( \pi(q, \omega_{ij} \mid x_{ij}) \) and \( \Pi(x, q, s_i^l \mid x) \) use expressions given in (19) and (22). Finally, that instead of the random variable \( \omega \), the assembler will face \( E\omega \) in all future states of the world.

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4Trading in futures contracts is organized so that futures contract defaults are completely avoided (for additional discussion, see §2–3 in Hull, 2009).
6.3 Comparison of Pass-Through and Revenue Sharing Contracts with Borrowing

Recall that in the benchmark case of Section 5 we show that the revenue sharing contract pareto dominates the pass-through contract. With financing, however, the above calculus changes as one of two things can happen to the revenue sharing contract: The stochastic input costs cause the revenue sharing contract to become infeasible, or the revenue sharing contract looses efficiency in situations where the pass-through contract does not. In fact, the efficiency loss can be so significant that it becomes possible for the pass-through contract to pareto dominate the revenue sharing contract, as will be seen. Taken together, the combination of volatile commodity input prices and operating debt therefore emerge as one possible explanation for the widespread use of the pass-through contract in practice.

6.3.1 An Infeasible Revenue Sharing Contract

To generate a case where the revenue sharing contract is infeasible, consider the problem of an upstream supplier with zero operating capital (i.e., \( y_s = 0 \)) wanting to revenue share with a downstream assembler with strictly positive operating capital (i.e., \( y_a > 0 \)).

To implement the revenue sharing contract, the supplier must borrow at time \( t_1 \), which requires \( y_s > 0 \) – a result, which is driven by the presence of stochastic costs in the model (cf. Lemma 4). However, because \( y_s = 0 \), then the revenue sharing contract is infeasible. In contrast, to implement the pass-through contract, the supplier must again borrow at time \( t_1 \), but now only the assembler’s working capital, \( y_a \), matters (cf. Lemma 2).

The logic is as follows. With the pass-through, the upstream supplier is borrowing at time \( t_1 \) and his loan will be repaid using funds that the assembler borrows at time \( t_2 \). Therefore the supplier’s ability to borrow at time \( t_1 \) hinges on the assembler’s ability to borrow at time \( t_2 \), which increases as \( y_a \) increases. In contrast, the revenue sharing contract requires that the supplier borrow twice – once at time at \( t_1 \) and then again at time \( t_2 \). Therefore with the revenue sharing contract, the supplier’s ability to borrow at time \( t_1 \) hinges on her own ability to borrow again at time \( t_2 \), which increases as \( y_s \) increases.

The next Result 5 is a direct consequence of Lemmas 2 and 4.

**Result 5.** Suppose \( y_a \geq \left( s_1 h - \frac{1 - \rho_2}{\rho_2} \right) q - \frac{y_s}{\rho_2} \) and \( 1 > \rho_2 \geq \frac{L_{s,2} - L_{s,1}}{L_{s,1}} \). Then the pass-through contract is always feasible. The revenue sharing contract is feasible whenever (17) holds.
6.3.2 The Supply Chain Efficiency Loss With Revenue Sharing Contracts

To generate the second case, we assume:

(a-1) Lemmas 2 and 4 hold for $\omega_{hh}$.

(a-2) Conditions (12) and (21), and Lemma 4 holds $\omega_{hh}$.

(a-3) $q \leq \bar{q}(\omega_{hh})$ (where $\bar{q}(\omega_{hh})$ is specified in Lemmas 1 and 5).

Taken together, these assumptions ensure that both firms will produce in all states of the world under both contracts. Incidentally, the conditions also imply that the firms will hedge as described in Sections 6.1.5 and 6.2.3. (The incentive to hedge comes from reduced expected financing costs.)

With these assumptions, we begin by showing that supply chain coordination with the revenue sharing contract breaks down.

**Result 6 (Lemma A.2).** Suppose (a-1) through (a-3) hold. There exist values $(\lambda_s^*, \lambda_a^*)$ such the revenue sharing contract of Section 6.2 is guaranteed to coordinate only when $\lambda_s = \lambda_s^*$ and $\lambda_a = \lambda_a^*$.

Result 6 is driven by working capital allocation in between the supplier and the assembler. To achieve full coordination, the both firms must be willing to re-allocate their working capital, which is often not practical. While the driving factor for the inefficiency for the pass-through contract with financing is that the assembler bears the entire demand risk, the driving factors for the efficiency loss of revenue sharing contracts are the misalignment between the working capital and revenue shares of supply chain parties. Surprisingly, there are many situations in which the inefficiencies of the revenue sharing contract tip the scales in favor of the pass-through contract.

**Result 7.** Suppose (a-1) through (a-3) hold. Then there exist a supplier’s profit threshold $\Pi^0$, a supplier’s working capital threshold $\bar{y}_s$, and an assembler’s working capital threshold $\bar{y}_a$ such that if the supplier requires to earn at least $\Pi^0$ profits, $y_s \leq \bar{y}_s$, and $y_a \geq \bar{y}_a$, then the pass-through contract of Section 6.1 pareto dominates the revenue sharing contract of Section 6.2.

Result 7 can be viewed as a trade-off between two evils: The evil of double marginalization and the evil of financing cost. Of course, in the no borrowing case, double marginalization caused by the fixed markup, $\gamma$, is the only source of inefficiency in the supply chain and the firms revenue share so as to avoid it. (Recall that in the no borrowing case the revenue sharing fully coordinates.) In the borrowing case, double marginalization is *unavoidable* because the ability of the revenue sharing
contract to coordinate breaks down (Result 6). As such, revenue sharing no longer has a clear advantage over pass-through and financing costs can decide which contract dominates the other.

To see that, consider that with the revenue sharing contract, the supplier earns $\lambda_s R(q)$ of the operating revenue and must borrow $L_{s,2} = \left[ \lambda_s (s_i^1 q + v_j^2 q) - y_s \right]^+$, $i \in \{l, h\}$, $j \in \{l, m, h\}$ at the cost of $r_{s,2}$ in order to cover her share of the operating costs. In order to match the supplier’s profit requirement, $\Pi^0$, $\lambda_s$ must be sufficiently high, which also increases the supplier’s borrowing cost $L_{s,2} \cdot r_{s,2}$. Therefore with the revenue-sharing contract, to earn more, the supplier incurs higher financing costs.

In contrast, with the pass-through contract – because the assembler hedges both commodities – the supplier borrows at the risk-free rate, which is zero (see Section 6.1.5) for a total payoff of $\gamma q$. To match the supplier’s profit requirement, $\Pi^0$, the quantity, $q$, must be sufficiently high, but higher $q$ does not impose any additional financing costs on the supplier. Therefore with the pass-through contract, the supplier can earn more without having to incur higher financing costs.

Now to see, how the revenue sharing contract “looses against” the pass-through contract, suppose $y_s$ is low relative to $y_a$. To match the supplier’s pass-through payout with the revenue sharing contract, the value of $\lambda_s$ must be sufficiently high, which means that the supplier incurs high borrowing costs, which cause the revenue sharing contract to become inefficient.

To see the flipside, suppose that the allocation of $y_s$ and $y_a$ are balanced. Then the revenue sharing contract wins because double marginalization – not financing costs – become the dominant source of inefficiency and revenue sharing with an a balanced split of the operating revenues beats the pass-through on double marginalization.

**Example 1.** We conduct the following numerical study: the demand distribution is assumed to be uniform $U[25, 175]$, the supplier’s working capital is $y_s = 100$, and the assembler’s working capital is $y_a = 1500$. Also, we let $\rho_1 = \rho_2 = 0.5$, $s_i^h = 11$ and $s_1^l = 5$, with a mean $s_1 = \rho_1 11 + (1 - \rho_1)5 = 8$, $v_2^h = 13$, $v_2^m = 10$ and $v_2^l = 7$, with a mean $v_2 = \rho_2 13 + 2 \rho_2 (1 - \rho_2) 10 + (1 - \rho_2) 7 = 10$. Then, we compare the performance difference between pass through contracts and revenue sharing contracts. For $(\lambda_s, \lambda_a) = (0.2, 0.8), (0.3, 0.7), (0.4, 0.6)$ and $(0.5, 0.5)$ four cases, our numerical results indicate:

- The assembler’s optimal production decision is to produce in all of the states $s_i^1, v_j^2$, for $i \in \{h, l\}$ and $j \in \{h, m, l\}$, i.e., no state that the assembler cannot borrow or get a negative expected profit, at the optimal production quantity;
○ Under the assembler’s optimal production quantity, the revenue sharing contracts (0.2, 0.8) and (0.3, 0.7) do not have the noticeable performance differences from the corresponding pass through contracts (i.e., when we let the supplier receives the same profit under the pass through contracts, the assembler’s profit is the same as under the revenue sharing contracts);

○ However, the revenue sharing contracts (0.4, 0.6) and (0.5, 0.5) do have the noticeable performance differences from the corresponding pass through contracts: Under the revenue sharing contract (0.4, 0.6), the assembler’s and supplier’s profits are 1100.87 and 728.68, respectively, while under the corresponding pass-through contract, they are 1155.70 and 728.79, respectively; under the revenue sharing contract (0.5, 0.5), the assembler’s and supplier’s profits are 917.39 and 909.93, respectively, while under the corresponding pass-through contract, they are 939.71 and 910.35, respectively. Of course, for the pass through contracts, here we do not optimize the assembler’s profit further under the supplier’s profit requirement constraint.

○ To see the flipside, consider $y_s = 100$ and $y_a = 400$. Under the revenue sharing contract (0.4, 0.6), the assembler’s and supplier’s profits are 1238.77 and 794.19, respectively, while under the corresponding pass-through contract, they are 1160.05 and 793.21, respectively.

### 7 Conclusion

Practicing managers are seriously challenged in effectively dealing with commodity price risks in global supply chains. What seems to be the common advice for minimizing risks in such settings is to use pass-through contracts. The logic of the pass-through arrangement is that upstream commodity risks are fully shifted to the buyer over the negotiated period of time. A buyer who fully hedges her exposure to all commodities involved in her products and services, when dealing with pass-through contracts with her suppliers, can offer the guarantee of a commodity risk free environment for her suppliers. However, a rational buyer does hold the option, figuratively and literally in the underlying mathematics, of not producing in the presence of very high commodity prices. Buyers are often protected by carefully negotiated penalties and limited liability laws if and when they default on such pass-through contracts. A buyer default breaks down the supplier risk free part of the argument behind such contracts. And, of course, immediately brings up the point of why such inefficient, non-coordinating contracts will be used in the first place within these risk fraught and decentralized supply chains.

Pursuing the argument of inefficiencies in decentralized supply chains in a frictionless world (no
taxes, need for financing, bankruptcy risk, information asymmetries etc.) brings us to asking the question of why not use appropriately adapted “revenue sharing” contracts (both revenues and costs are proportionately shared between supplier and buyer) to eliminate double marginalization inefficiencies in this environment. In a frictionless world, the revenue sharing contract is the ideal contract that eliminates inefficiencies and fully dominates any non-zero margin pass-through contract of these settings. Then what might be a plausible supporting logic for the predominance of pass-through contracts in commodity risk environments, going beyond monitoring and administration costs that the revenue sharing might impose, and which are getting less important in technologically connected chains? To offer this logic we depart from the frictionless world, and our friction of choice is limited working capital of each firm necessitating financing for their corresponding production and delivery needs. Such financing, even in the presence of non-strategic players competitively pricing the risks (e.g., idealized banking institutions) exposes firm to bankruptcy risks. With financing frictions in place, both the pass-through contract and the revenue sharing one are non-coordinating and suffer from double marginalization inefficiencies.

The presence of financing frictions makes the supporting logic of the pass-through contract stronger. The supplier attempts to shift all the commodity risks of their purchases to the downstream buyer, who now has more incentives than before to hedge. If the buyer has fully hedged the commodity risks in the supply chain, not only she lowers her financing costs but also helps the supplier borrow at a low, risk free rate. In the absence of hedging, costly financing might reduce the quantities that the upstream supplier can finance and therefore produce, which limits the profits of the buyer. Eliminating such an upstream “bottleneck” is in many cases enough to incentivize the buyer to hedge, and this way completes the supporting logic of the pass-through contract: creating a commodity risk free supply environment for the upstream firm and a functioning supply chain for the downstream buyer in all states of the world. In contrast, the presence of financing frictions weakens the logic of the use for revenue sharing contract in this volatile commodity price setting. The nature of the contract requires sharing of all commodity costs as early as any production happens, which is earlier than usual for the buyer (as compared with using a wholesale price or a pass-through contract) and for indirect commodity purchases not of her usual concern. At the same time, it requires participation in downstream costs for commodities of not of his usual concern, and having to support these costs for longer than his usual time frame. As a result, the financing needs of both supplier and buyer are
more substantial, and required over a longer time period for both of them. The financing needs of a revenue sharing contract for limited working capital firms, especially suppliers, may make infeasible the implementation of the revenue sharing contract in some cases. Furthermore, failure of either firm to hedge increases not only their financing costs but also those of their counterpart, but hedging itself is not enough to guarantee elimination of double marginalization inefficiencies in a decentralized supply chain with firms of differential working capital levels and financing needs over time. In the absence of the right level of working capital of the downstream buyer, financing create “bottlenecks” in the chain (desired quantities to be produced may be constrained by inadequate working capital support for purchasing commodity inputs). In a financial frictions fraught supply chain, there are clear trade-offs between increased supply chain efficiency, via producing the appropriate quantities to lower double marginalization, and increased financing costs to provide the needed working capital support. And as result, there are environments that the revenue sharing contract suffers inefficiencies and financing costs that allow the pass-through contract to dominate it.

What appears to be the “sweet spot” of an environment for application of the pass-through contract is a supply chain of a supplier with limited working capital (indicating either financial weakness or lack of willingness to commit substantial capital towards the supply chain transaction), with expectations of high profit threshold for participating in the transaction (indicative of either high margins or power within the chain), and dealing with a buyer of reasonable working capital availability. Instead of thinking of the pass-through contract as an effective commodity risk minimization contract, we argue one should think of it as the most effective financing contract for this setting: a well-to-do buyer assumes the commodity risks of the chain, and through hedging reduces her financing costs while at the same time allowing their under-capitalized supplier to borrow at a risk free rate. Low working capital of the supplier can limit the use of the revenue sharing contract due to the increase in financing needs, and at the same time the misallocated working capital within the chain for the financing needs of the intended production quantity drive up financing costs of all players and create double-marginalization inefficiencies relative to the first-best solution.

References


Appendix A  Breakdown Of Supply Chain Coordination With Revenue Sharing

A variant of this result with deterministic cost has been put forth by Kouvelis and Zhao, 2016.

**Lemma A.1.** Suppose (a-1) through (a-3) hold. Then the firms’ expected profits with revenue sharing can be written as:

\[
\Pi(q, E_\omega; \lambda_c, y_c) = \lambda_c [S(q) - E_\omega q - C(k_c(\lambda_c, y_c), q)], \quad \text{for } c = s, \tag{A.1}
\]

\[
\pi(q, E_\omega; \lambda_c, y_c) = \lambda_c [S(q) - E_\omega q - C(k_c(\lambda_c, y_c), q)], \quad \text{for } c = a, \tag{A.2}
\]

where the expected loss function \(C\) is defined as

\[
C(k_c, q) = \begin{cases} 
\alpha \frac{1}{2} \int_0^{2k_c} \xi^2 f(\xi) d\xi, & k_c \leq q; \\
\alpha \left[ \frac{1}{4} \int_0^q \xi^2 f(\xi) d\xi + \int_{2q-k_c}^{2q} (\xi - q) f(\xi) d\xi \right], & k_c > q, 
\end{cases} \tag{A.3}
\]

and where \(k_c\) is defined through

\[
S_b(k_c, q) = L_c(q) = \left[ E_\omega q - \frac{y_c}{\lambda_c} \right]^+ \tag{A.4}
\]

for \(c \in \{s, a\}\) (see Equation (B.1) for the definition of function \(S_b\)).

From Lemma A.1, each firm’s expected profit can be viewed as composed of two parts: \((S(q) - q E_\omega)\) and \(C(k_c(\lambda_c, y_c), q)\). The first part is independent of the revenue share \(\lambda_c\), and can be perfectly split between the supplier and assembler. However, the second part \(C(k_c(\lambda_c, y_c), q)\) is a function of \(\lambda_c\), and this part implies that the total profits of the supplier and assembler will change with the relative values of \(\lambda_s\) and \(\lambda_a\), even if the ordering quantity \(q\) does not change. Our next result establishes how the total supply chain profit changes in the revenue sharing contract \((\lambda_s, \lambda_a)\). Let \(q^*(\lambda_s, \lambda_a)\) denote the assembler’s optimal ordering quantity under the contract \((\lambda_s, \lambda_a)\).

**Lemma A.2.** Suppose (a-1) and (a-2) hold. For IFR demand distributions and under a revenue sharing contract \((\lambda_s, \lambda_a)\),

1. For any given \(q \geq 0\),

\[
\min_{\lambda_s, y_s; \lambda_a, y_a} \lambda_s C(k_s(\lambda_s, y_s), q) + \lambda_a C(k_a(\lambda_a, y_a), q)
\]

is achieved if \(\lambda_s, y_s, \lambda_a\), and \(y_a\) satisfy the following condition:

\[
\frac{y_s}{\lambda_s} = \frac{y_a}{\lambda_a} = y_s + y_a. \tag{A.5}
\]

The condition is also necessary if \(q > \frac{y_s + y_a}{E_\omega}\).
(2) Let \((\lambda_s^\ast, \lambda_a^\ast)\) be the revenue sharing contract corresponding to condition (A.5). Then,

\[
\Pi(q^\ast(\lambda_s, \lambda_a), \omega; \lambda_s, y_s) + \pi(q^\ast(\lambda_s, \lambda_a), \omega; \lambda_a, y_a) \leq \\
\Pi(q^\ast(\lambda_s^\ast, \lambda_a^\ast), \omega; \lambda_s^\ast, y_s) + \pi(q^\ast(\lambda_s^\ast, \lambda_a^\ast), \omega; \lambda_a^\ast, y_a).
\] (A.6)

That is, the supply chain’s total profit is maximized at the contract \((\lambda_s^\ast, \lambda_a^\ast)\). Particularly, if \(q^\ast(\lambda_s^\ast, \lambda_a^\ast) > \frac{y_s + y_a}{E}\), then the equality holds in (A.6) only for \(\lambda_s = \lambda_s^\ast\) and \(\lambda_a = \lambda_a^\ast\).

Lemma A.2 is driven by working capital allocation. If the supply chain members do not want to re-allocate their working capitals, a revenue sharing contract can fail to coordinate the supply chain. Particularly, \(q^\ast(\lambda_s^\ast, \lambda_a^\ast) > \frac{y_s + y_a}{E}\) implies that the supply chain optimal production quantity is high so that even under the optimal revenue sharing contract \((\lambda_s^\ast, \lambda_a^\ast)\), the supply chain will generate some expected bankruptcy loss. Thus, if a contract \((\lambda_s, \lambda_a) \neq (\lambda_s^\ast, \lambda_a^\ast)\), the expected bankruptcy loss becomes larger. Therefore, \((\lambda_s^\ast, \lambda_a^\ast)\) is the only coordinating contract.

Appendix B Proofs

Proof of Lemma 1. We begin by summarizing the bank’s expected revenue, \(S_b(k, q)\), as follows:

\[
S_b(k, q) = \begin{cases} 
\frac{1}{4} f_0^{2k}(1 - \alpha) \xi^2 f(\xi)d\xi + k^2 f_2^\infty h(\xi)d\xi, & k \leq q; \\
\frac{1}{4} f_0^{2q}(1 - \alpha) \xi^2 f(\xi)d\xi + f_2^{2k} (1 - \alpha) q(\xi - q)f(\xi)d\xi + q(2k - q) F(2k), & k > q.
\end{cases}
\] (B.1)

Observe that

\[
\frac{\partial S_b(k = k_a, q)}{\partial k} = \begin{cases} 
2k F(2k) G(k, q), & k \leq q, \\
2q F(2k) G(k, q), & k > q,
\end{cases}
\]

where \(G(k, q) = \begin{cases} 
1 - \alpha kh(2k), & k \leq q, \\
1 - \alpha (2k - q) h(2k), & k > q.
\end{cases}\) (B.2)

where \(h(\cdot) = \frac{f(\cdot)}{F(\cdot)}\) is the hazard rate of the market potential, \(\xi\). Note that increasing hazard rate\(^5\) implies \(\partial S_b(k_a, q)/\partial k_a \leq 0\). Moreover, with increasing hazard rate, we have that \(S_b(k_a, q)\) achieves its optimal value at \(\xi = 2\bar{k}_a\), where \(\bar{k}_a\) solves \(G(\bar{k}_a, q) = 0\). Then for a given \(q \geq 0\), the assembler has a borrowing capacity, which is achieved at \(2\bar{k}_a\). This means that the bank will only accept loans with \(k_a \leq \bar{k}_a\).

Note that \((\omega_{ij} q - y_a)\) increases in \(q\). We then focus on the case that \((\omega_{ij} q - y_a) > 0\) so that the assembler needs to borrow. Define a function \(H(q)\) as follows:

\[
H(q) = S_b(\bar{k}_a(q), q) - \omega_{ij} q + y_a.
\]

Note that if \(H(q) < 0\), then the bank cannot break-even. So, we need \(q\)’s for which \(H(q) \geq 0\).

\[
H'(q) = \frac{\partial S_b(\bar{k}_a(q); \alpha)}{\partial \bar{k}_a} \cdot \frac{\partial \bar{k}_a}{\partial q} + \frac{\partial S_b(\bar{k}_a(q); \alpha)}{\partial q} - \left( s_1^1 + v_2^j + \gamma \right)
\]

\[
= \frac{\partial S_b(\bar{k}_a(q); \alpha)}{\partial q} - \frac{S_b(\bar{k}_a(q); \alpha) + y_a}{q}
\]

\(^5\)IFR distributions have increasing hazard rates – for further details, see Lariviere (2006).
which implies that 2
\frac{\partial k}{\partial q} where 
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If 
P
x
First, suppose 
\frac{\partial \pi}{\partial q} = 0. Then 
S
(2
\frac{\partial q}{\partial \omega} = \pi - a
x
\bar{\omega}
1
\frac{\partial S_t(k_a, q)}{\partial k_a} = \frac{S_t(k_a, q) + y_a}{q} - \frac{\partial S_t(k_a, q)}{\partial q}
\frac{\partial (2k_a - q)}{\partial q} = \frac{1}{2q} \int_0^q (1 - \alpha) \xi^2 f(\xi) d\xi + 2(1 - \alpha) q \frac{f(\xi)}{2q} f(\xi) d\xi + \frac{\partial F(2k_a)}{\partial q} + \frac{2y_a}{q} \geq 0,
\frac{\partial (2k_a - q)}{\partial q} = \frac{1}{2q} \int_0^q (1 - \alpha) \xi^2 f(\xi) d\xi + 2(1 - \alpha) q \frac{f(\xi)}{2q} f(\xi) d\xi + \frac{\partial F(2k_a)}{\partial q} + \frac{2y_a}{q} - 1
\frac{\partial \pi}{\partial q} = \frac{S_t(k_a, q) + y_a}{q} - \frac{\partial S_t(k_a, q)}{\partial q}
\frac{\partial (2k_a - q)}{\partial q} \equiv \frac{1}{2q} \int_0^q (1 - \alpha) \xi^2 f(\xi) d\xi + 2(1 - \alpha) q \frac{f(\xi)}{2q} f(\xi) d\xi + \frac{\partial F(2k_a)}{\partial q} + \frac{2y_a}{q} \geq 0,
\frac{\partial \pi}{\partial q} = \frac{S_t(k_a, q) + y_a}{q} - \frac{\partial S_t(k_a, q)}{\partial q}
\frac{\partial (2k_a - q)}{\partial q} = \frac{1}{2q} \int_0^q (1 - \alpha) \xi^2 f(\xi) d\xi + 2(1 - \alpha) q \frac{f(\xi)}{2q} f(\xi) d\xi + \frac{\partial F(2k_a)}{\partial q} + \frac{2y_a}{q} \geq 0,
which implies that 2\frac{\partial k}{\partial q} \geq 1.

**Proof of Lemma 3.** If \( x_{ij} = 0 \), then \( \pi(q, \omega_{ij} \mid 0) \) does not depend on \( \omega_{ij} \) and the result follows. If \( x_{ij} = 1 \), then Equations (10) imply:
\[
\frac{\partial \pi}{\partial q} = \frac{\partial \pi}{\partial \omega_{ij}} \cdot \frac{\partial \omega_{ij}}{\partial q} + \frac{\partial \pi}{\partial q} \leq 0.
\]
To establish the inequality, we used Lemma 1 and the fact that
\[
\frac{\partial S_t(k_a, q)}{\partial k_a} = \begin{cases} \frac{2k_a \bar{F}(2k_a)}{2q \bar{F}(2k_a)} & \text{if } k_a \leq q, \\ \frac{\partial S_t(k_a, q)}{\partial k_a} & \text{if } k_a > q, \end{cases}
\]
where \( S_t(k_a, q) \) is given by (11).

**Lemma B.1.** Suppose the distribution of the market potential, \( \xi \), is IFR and \( P_a(q) \) is convex in \( q \). Then \( \pi(q, \omega_{ij} \mid x) \), where \( \pi(q, \omega_{ij} \mid x_{ij}) \) is given by (10), is concave in \( q \) for all \( q \leq \bar{q}(\omega_{ij}) \).

**Proof of Lemma B.1.** We show that \( \pi(q, \omega_{ij} \mid x_{ij}) \), \( x \in \{0, 1\} \) is concave in \( q \). Then the result holds for the expectation as well because linear combinations of concave functions are concave.

First, suppose \( x = 0 \). Then \( \pi(q, \omega_{ij} \mid 0) = - \min \{y_a, P_a\} \), which is concave due to our assumptions about \( P_a \). Next, consider \( \pi(q, \omega_{ij} \mid 1) \). The first order condition is
\[
S'(q) - \frac{\partial S_t}{\partial k_a} \cdot \frac{\partial k_a}{\partial q} - \frac{\partial S_t}{\partial q} + \frac{\partial \min \{q \omega_{ij}, y_a\}}{\partial q}
= S'(q) - \frac{\partial S_t}{\partial k_a} \cdot \omega_{ij} - \frac{\partial S_t}{\partial q} - \frac{\partial S_t}{\partial q} = S'(q) - \frac{\omega_{ij} - \frac{\partial S_t}{\partial q}}{G(k_a, q)} - \frac{\partial S_t}{\partial q}
= S'(q) - \frac{\omega_{ij} - \frac{\partial S_t(k_a, q)}{\partial q}}{G(k_a, q)} + \frac{\partial A(k_a, q)}{\partial q} - \frac{\partial S_t(k_a, q)}{\partial q}.
\]
where $G(k_a, q)$ is defined in (B.2).

For simplicity, let $B(k_a, q) = - \frac{\partial S_t(k_a, q)}{\partial q} + \alpha A(k_a, q)$. Note that

$$d \left[ \frac{\omega_{ij} + B(k_a, q)}{G(k_a, q)} + \frac{\partial S_t(k_a, q)}{\partial q} \right]$$

$$= \frac{1}{G(k_a, q)} \left[ \frac{dB(k_a, q)}{dq} - \omega_{ij} + B(k_a, q) \right] \cdot dG(k_a, q) + \frac{dS_t(k_a, q)}{dq}$$

$$= \frac{1}{G(k_a, q)} \left[ (- \partial^2 S_t(k_a, q) + \alpha A(k_a, q)) \frac{\partial k_a}{\partial q} + \left( - \frac{\partial^2 S_t(k_a, q)}{\partial q^2} + \alpha \frac{\partial A(k_a, q)}{\partial q} \right) \right]$$

$$\frac{1}{G^2(k_a, q)} \left[ \omega_{ij} + B(k_a, q) \right] \left[ \frac{\partial G(k_a, q)}{\partial q} \frac{\partial k_a}{\partial q} + \frac{\partial G(k_a, q)}{\partial q} \right] + \frac{\partial^2 S_t(k_a, q)}{\partial q^2} G^2(k_a, q)$$

$$= \frac{\partial k_a}{\partial q} \frac{G^2(k_a, q)}{G(k_a, q)}$$

$$\frac{\partial k_a}{\partial q} \left( - \frac{\partial^2 S_t(k_a, q)}{\partial q^2} + \alpha \frac{\partial A(k_a, q)}{\partial q} \right) G(k_a, q) - \omega_{ij} + B(k_a, q) + \frac{\partial G(k_a, q)}{\partial q} \frac{\partial^2 S_t(k_a, q)}{\partial q^2} G^2(k_a, q)$$

$$\frac{\partial k_a}{\partial q} \left( 2F(2k_a)(1 - G(k_a, q)) + \frac{\partial A(k_a, q)}{\partial q} \right) G(k_a, q) - \omega_{ij} + B(k_a, q) + \frac{\partial G(k_a, q)}{\partial q} \frac{\partial^2 S_t(k_a, q)}{\partial q^2} G^2(k_a, q)$$

$$\geq \frac{\alpha}{G^2(k_a, q)} \left\{ \frac{\partial k_a}{\partial q} \left[ -qF(2k_a)h(2k_a)G(k_a, q) + [\omega_{ij} + B(k_a, q)]h(2k_a) + (2k_a - q)h'(2k_a) \right] \right\}$$

$$\geq \frac{\alpha}{G^2(k_a, q)} \left\{ \frac{\partial k_a}{\partial q} \left[ -qF(2k_a)h(2k_a)G(k_a, q) + [\omega_{ij} + B(k_a, q)]h(2k_a) \right] \right\}$$

$$= \frac{\partial k_a}{\partial q} \left[ -qF(2k_a)h(2k_a)G(k_a, q) + [\omega_{ij} + B(k_a, q)]h(2k_a) \right]$$

We now show that $2F(2k_a)(2k_a - q)h(2k_a) - 2F(2k_a)h(2k_a) \geq 0$ for $k_a \geq q$. Let $H(x) = 2F(2k_a)(2k_a - q)h(2k_a) - 2F(2k_a)h(2k_a)$. Then, $H(q) = 0$. Also, $H'(x) = 2F(2k_a)(2k_a - q)h(2k_a) - 2F(2k_a)h(2k_a)$.
Proof of Result 3. If (i) and (ii) hold, then \( x_{ij} = 1 \) for all \( i, j \in \{l, h\} \) and \( j \in \{l, m, h\} \). Then Lemma B.2 and Jensen’s inequality then imply:

\[
\mathbb{E} \pi(q, \omega | x) \leq \pi(q, \mathbb{E} \omega | x),
\]

(8B)

Note that the assembler achieves \( \pi(q, \omega_i \mid x_{ij} = 1) \) by purchasing \( n_{1,1} = n_{2,2} = q \) futures contracts at time \( t_0 \). Since \( n_{1,1} = n_{2,2} = q \), wipes out the noise in the supplier’s payoff, then his forward market position is irrelevant (i.e., the supplier does not need to hedge). □

Lemma B.2. Suppose the distribution of the market potential, \( \xi \), is IFR. Then \( \pi(q, \omega_i \mid x_{ij} = 1) \), given by (10), is concave in \( \omega_{ij} \).

Proof of Lemma B.2. Let \( L_a = (\omega_i q - y_a)^+ \). If \( L_a = 0 \), then we have \( k_a = 0 \) so that \( S_t(k_a, q) = 0 \) and the conclusion holds. Consequently, we focus on the case \( L_a > 0 \). From Equation (7),

\[
\frac{\partial k_a}{\partial L_a} = \frac{1}{\frac{\partial S_t(k_a, q)}{\partial k_a}}.
\]

Then, we have

\[
\frac{\partial S_t(k_a, q)}{\partial L_a} = \frac{\partial S_t(k_a, q)}{\partial k_a} \cdot \frac{\partial k_a}{\partial L_a} = \frac{\partial S_t(k_a, q)}{\partial k_a} \cdot \frac{1}{G(k_a, q)} \cdot \frac{1}{\frac{\partial S_t(k_a, q)}{\partial k_a}} = \frac{1}{G(k_a, q)},
\]

where the second equality follows from Equation (B.5). Since \( G(k_a, q) \geq 0 \), we have \( \frac{\partial S_t(k_a, q)}{\partial L_a} \geq 0 \). (For definition of the function \( G \), see Equation B.2). □

Proof of Lemma A.1. Note that the loan that firm \( c, c \in \{s, a\} \), borrows from the bank is then \( L_c(q) = [\lambda_c \mathbb{E} \omega q - y_c]^+ \). According to the bank’s expected revenue \( S_b \) defined in Equation (B.1), at the end of the sales season, what the bank expects to receive from firm \( c \) is \( \lambda_c S_b(k_c, q) \). Then, the bank will determine the interest rate so that the corresponding bankruptcy threshold \( k_c \) satisfies \( \lambda_c S_b(k_c, q) = [\lambda_c \mathbb{E} \omega q - y_c]^+ \), which is Equation (A.4). Through the equation, \( k_c \) is defined as a function of \( \lambda_c \) and \( y_c \).

From Equation (11), and with the definition of the function \( C \) in Equation (A.3), firm \( c \)'s expected cost, in the case of borrowing, can be rewritten as

\[
\lambda_c S_t(k_c, q) = \lambda_c [S_b(k_c, q) + C(k_c(\lambda_c, y_c), q)] = \lambda_c \left[ \left( \mathbb{E} \omega q - \frac{y_c}{\lambda_c} \right)^+ + C(k_c(\lambda_c, y_c), q) \right].
\]

On the other hand, if the firm does not borrow, then his cost is \( \min\{\lambda_c \mathbb{E} \omega q, y_c\} \). Note that \( \lambda_c \left( \mathbb{E} \omega q - \frac{y_c}{\lambda_c} \right)^+ + \min\{\lambda_c \mathbb{E} \omega q, y_c\} = \lambda_c \mathbb{E} \omega q \). Consequently, his expected profit is

\[
\lambda_c S(q) - \lambda_c S_t(k_c, q) - \min\{\lambda_c \mathbb{E} \omega q, y_c\} = \lambda_c [S(q) - \mathbb{E} \omega q - C(k_c(\lambda_c, y_c), q)].
\]

Then, we obtain Equations (A.1) and (A.2). □
\textbf{Proof of Lemma A.2.} We first show Part 1. Consider the case \( 0 \leq q \leq \frac{y_s + y_a}{\varepsilon \omega} \). If condition (A.5) is satisfied, then from Equation (A.4), we have

\[
S_b(k, q) = \left[ \varepsilon \omega q - \frac{y_c}{\lambda_c} \right]^+ = [\varepsilon \omega q - (y_s + y_a)]^+ = 0.
\]

Therefore, \( k_s(\lambda_s, y_s) = k_a(\lambda_a, y_a) = 0 \). From Equation (A.3), we have \( C(k_s(\lambda_s, y_s), q) + C(k_a(\lambda_a, y_a), q) = 0 \), and the conclusion holds.

Now, we study the case \( q > \frac{y_s + y_a}{\varepsilon \omega} \). For ease of exposition, we use the generic parameter \( \lambda, k, \) and \( y \) in the corresponding functions. Also, since \( q \) is given, we regard \( q \) as a constant instead of a variable, and use \( C(\lambda, y) \) to represent \( C(k(\lambda, y), q) \) for simplicity. Let \( H(\lambda, y) = \lambda C(\lambda, y) \). From Equation (A.4), \( k(\lambda, y) \) is only a function of \( \frac{y}{\lambda} \). As a result, \( k(\theta \lambda, \theta y) = k(\lambda, y) \) for any \( \theta > 0 \). From Equation (A.3),

\[
H(\theta \lambda, \theta y) = \begin{cases}
\theta \lambda \cdot \alpha \frac{1}{4} \int_0^{2k(\theta \lambda, \theta y)} \frac{\xi^2 f(\xi)}{f(\xi)} d\xi, & k(\theta \lambda, \theta y) \leq q; \\
\theta \lambda \cdot \alpha \left[ \frac{1}{4} \int_0^{2k(\theta \lambda, \theta y)} \frac{\xi^2 f(\xi)}{f(\xi)} d\xi + \int_{2q}^{2k(\theta \lambda, \theta y)} q(\xi - q) f(\xi) d\xi \right], & k(\theta \lambda, \theta y) > q.
\end{cases}
\]

As a result, function \( H(\lambda, y) \) is positively homogeneous in \( \lambda \) and \( y \) of degree 1. Then, \( H(\lambda, y) \) is sub-additive in \( \lambda \) and \( y \), if and only if \( H \) is a joint convex function of the two variables.

From (A.4), we have

\[
\frac{\partial k}{\partial y} = \frac{1}{\lambda \frac{\partial S_b(k, q)}{\partial k}} \quad \text{and} \quad \frac{\partial k}{\partial \lambda} = -\frac{S_b(k, q)}{\lambda \frac{\partial S_b(k, q)}{\partial k}}.
\]

As a result,

\[
\frac{\partial H(\lambda, y)}{\partial y} = \begin{cases}
\frac{\alpha k^2 F(2k) h(2k)}{\frac{\partial S_b(k, q)}{\partial k}}, & k \leq q; \\
\frac{\alpha q(2k-q) F(2k) h(2k)}{\frac{\partial S_b(k, q)}{\partial k}}, & k > q,
\end{cases} \quad \frac{\partial H(\lambda, y)}{\partial \lambda} = \begin{cases}
\frac{\alpha k h(2k)}{1 - \alpha k h(2k)}, & k \leq q; \\
\frac{\alpha(2k-q) h(2k)}{1 - \alpha(2k-q) h(2k)}, & k > q,
\end{cases}
\]

where \( G(k, q) \) is defined in Equation (B.2). Therefore,

\[
\begin{align*}
\frac{\partial^2 H(\lambda, y)}{\partial y^2} & = -\frac{G'(k)}{G^2(k)} \cdot \frac{1}{\lambda \frac{\partial S_b(k, q)}{\partial k}}, \\
\frac{\partial^2 H(\lambda, y)}{\partial y \partial \lambda} & = -\frac{G'(k)}{G^2(k)} \cdot \frac{S_b(k, q)}{\lambda \frac{\partial S_b(k, q)}{\partial k}} = -S_b(k, q) \frac{\partial^2 H(\lambda, y)}{\partial y^2}, \\
\frac{\partial^2 H(\lambda, y)}{\partial \lambda^2} & = -\frac{H(\lambda, y)}{\lambda^2} + \frac{1}{\lambda} \cdot \frac{\partial H(\lambda, y)}{\partial \lambda} - \frac{\partial S_b(k, q)}{\partial k} \cdot \frac{\partial k}{\partial \lambda} \cdot \frac{\partial H(\lambda, y)}{\partial y} - S_b(k, q) \frac{\partial^2 H(\lambda, y)}{\partial y \partial \lambda}.
\end{align*}
\]
Also, note that \( \lambda \) where equality holds if and only if condition \((A.5)\) holds.

From Equation \((B.15a)\),
\[
\partial H(\lambda, y) = S_b(k, q) \frac{\partial^2 H(\lambda, y)}{\partial y \partial \lambda}.
\]

Then, the Hessian matrix of function \( H(\lambda, y) \) is
\[
\begin{bmatrix}
\frac{\partial^2 H(\lambda, y)}{\partial \lambda^2} & \frac{\partial^2 H(\lambda, y)}{\partial \lambda \partial y} \\
\frac{\partial^2 H(\lambda, y)}{\partial \lambda \partial y} & \frac{\partial^2 H(\lambda, y)}{\partial y^2}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial^2 H(\lambda, y)}{\partial y^2} & -S_b(k, q) \frac{\partial^2 H(\lambda, y)}{\partial y^2} \\
-S_b(k, q) \frac{\partial^2 H(\lambda, y)}{\partial y^2} & S_b(k, q) \frac{\partial^2 H(\lambda, y)}{\partial y^2}
\end{bmatrix}
\]
\[
\cong
\begin{bmatrix}
\frac{\partial^2 H(\lambda, y)}{\partial y^2} & 0 \\
0 & 0
\end{bmatrix},
\]

where \( A \cong B \) means that matrices \( A \) and \( B \) have the same positive/negative semi-definite properties. Consequently, the Hessian matrix is positive semi-definite if and only if \( \frac{\partial^2 H(\lambda, y)}{\partial y^2} \geq 0 \), i.e., \( H(\lambda, y) \) is convex in \( y \) for a given \( \lambda \). Given that \( H(\theta \lambda, \theta y) = \theta H(\lambda, y) \) for any \( \theta > 0 \), we have \( H(\lambda, y) \) is sub-additive in \( y \) and \( \lambda \) if and only if \( H(\lambda, y) \) is convex in \( y \) for a given \( \lambda \).

From Equation \((B.12a)\),
\[
\frac{\partial H(\lambda, y)}{\partial y} = \begin{cases} 
\frac{1}{1-\alpha kh(2k)} - 1, & k \leq q; \\
\frac{1}{1-\alpha(2k-q)h(2k)} - 1, & k > q,
\end{cases} \quad (B.15a)
\]

Note that \( 1 - \alpha[k + (k - q)^+]h(2k) \geq 0 \) and decreases in \( k \). Then, \( \frac{\partial H(\lambda, y)}{\partial y} \) is continuous and increases in \( k \). From Equation \((B.11)\), \( k \) increases in \( y \). Then, \( \frac{\partial H(\lambda, y)}{\partial y} \) increases in \( y \) monotonically so that \( H(\lambda, y) \) is convex in \( y \) for a given \( \lambda \). Consequently, \( H(\lambda, y) \) is jointly convex in \( y \) and \( \lambda \), and thus it is subadditive in the two variables. That is, for \( \lambda_s \) and \( \lambda_a \) so that \( E\omega q - \frac{\chi_s}{\lambda_s} \geq 0 \) and \( E\omega q - \frac{\chi_a}{\lambda_a} \geq 0 \), we have
\[
H(\lambda_s, y_s) + H(\lambda_a, y_a) \geq H(\lambda_s + \lambda_a, y_s + y_a) = H(1, y_s + y_a),
\]
where equality holds only if \( \frac{\chi_s}{\lambda_s} = \frac{\chi_a}{\lambda_a} = y_s + y_a \). This is condition \((A.5)\).

If at least one \( E\omega q - \frac{\chi_s}{\lambda_s} \) or \( E\omega q - \frac{\chi_a}{\lambda_a} \) is not positive, without loss of generality, we assume \( E\omega q - \frac{\chi_s}{\lambda_s} < 0 \). Then, \( \lambda_s E\omega q - y_s > E\omega q - (y_s + y_a) > 0 \). As a result, \( H(\lambda_s, y_s) + H(\lambda_a, L_a) > H(1, y_s + y_a) \), since from Equation \((B.15a)\), \( \frac{\partial H(\lambda, y)}{\partial y} > 0 \) for \( 0 \leq k < \tilde{k}(q) \). Thus, \( H(\lambda_s, y_s) + H(\lambda_a, y_a) \geq H(1, y_s + y_a) \), where equality holds if and only if condition \((A.5)\) holds.

We next show Part 2. From Part 1 and Equations \((A.1)\) and \((A.2)\), for any give \( q \geq 0 \), we have
\[
\Pi(q, E\omega; \lambda_s, y_s) + \pi(q, E\omega; \lambda_a, y_a) \leq \Pi(q, E\omega; \lambda^*_s, y_s) + \pi(q, E\omega; \lambda^*_a, y_a).
\]

Also, note that \( \lambda^*_s \) and \( \lambda^*_a \) do not depend on \( q \). Then, for any revenue sharing contract \((\lambda_s, \lambda_a)\),
\[
\Pi(q^*(\lambda_s, \lambda_a), E\omega; \lambda_s, y_s) + \pi(q^*(\lambda_s, \lambda_a), E\omega; \lambda_a, y_a)
\leq \Pi(q^*(\lambda_s, \lambda_a), E\omega; \lambda^*_s, y_s) + \pi(q^*(\lambda_s, \lambda_a), E\omega; \lambda^*_a, y_a)
\leq \Pi(q^*(\lambda^*_s, \lambda^*_a), E\omega; \lambda^*_s, y_s) + \pi(q^*(\lambda^*_s, \lambda^*_a), E\omega; \lambda^*_a, y_a),
\]

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which is inequality (A.6). The result for the case that $q^\ast(\lambda^*_s, \lambda^*_a) > \frac{y_a + \gamma y_a}{E}$ holds directly from the Part 1 of the proposition.

\begin{proof}

Proof of Result 7. We first consider a given $q$. Under a pass-through contract, the supplier’s profit will be $q\gamma$. The assembler’s profit is $\pi(1, q, \mathbb{E}\omega) = S(q) - S_t(k_a, q) - \min\{(\mathbb{E}\omega + \gamma)q, y_a\}$ from Equation (10).

As in Lemma A.1, we can rewrite the assembler’s profit as

$$\pi(1, q, \mathbb{E}\omega) = S(q) - (\mathbb{E}\omega + \gamma)q - C(k_a(\mathbb{E}\omega + \gamma, y_a), q),$$

where the function $C(k_a, q)$ is defined in Equation (A.3) and where $k_a(\mathbb{E}\omega + \gamma, y_a)$ is defined through

$$S_b(k_a, q) = L_a(q) = [(\mathbb{E}\omega + \gamma)q - y_a]^+.$$

The total profit of the supply chain is then $q\gamma + \pi(1, q, \mathbb{E}\omega) = S(q) - \mathbb{E}\omega q - C(k_a(\mathbb{E}\omega + \gamma, y_a), q)$. 

We now consider a revenue sharing contract $(\lambda_s, \lambda_a)$. From Equations (A.1) and (A.2), we have the total supply chain profit

$$\Pi(q, \mathbb{E}\omega; \lambda_s, y_s) + \pi(q, \mathbb{E}\omega; \lambda_a, y_a) = S(q) - \mathbb{E}\omega q - [\lambda_s C(k_s(\lambda_s, y_s), q) + \lambda_a C(k_a(\lambda_a, y_a), q)].$$

Comparing the total supply chain profits under the pass-through contract and the revenue sharing contract, we can see if

$$C(k_a(\mathbb{E}\omega + \gamma, y_a), q) \leq \lambda_s C(k_s(\lambda_s, y_s), q) + \lambda_a C(k_a(\lambda_a, y_a), q),$$

then it is possible that under the pass-through contract, the total supply chain profit is larger than that under the revenue sharing contract.

To see such situation can exist for sure, we consider the situation the supplier has the requirement on the profit margin, i.e., $\Pi \geq \Pi^0 > 0$. Under the revenue sharing contract, let the supplier’s profit be exactly $\Pi^0$, i.e.,

$$\Pi(q, \mathbb{E}\omega; \lambda_s, y_s) = \lambda_s \left[ S(q) - \mathbb{E}\omega q - C(k_s(\lambda_s, y_s), q) \right] = \Pi^0.$$

Then, $\lambda_s$ is no smaller than a lower bound $\bar{\lambda}_s > 0$. Now, if the supplier’s working capital amount is small, i.e., $y_s \leq \bar{y}_s$, then from Equation (A.4), we have $k_s(\lambda_s, y_s) > 0$ and thus $\lambda_s C(k_s(\lambda_s, y_s), q) > 0$.

Under the pass-through contract, for the supplier to receive exactly $\Pi^0$ profits, the supplier’s profit margin has to be $\gamma = \frac{\Pi^0}{q}$, or equivalently, $q \gamma = \Pi^0$.

Now, we assume that the assembler’s working capital amount is large, i.e., $y_a \geq \bar{y}_a \geq (\mathbb{E}\omega + \gamma)q$. (This is only a sufficient condition, and we might not require $\bar{y}_a$ to be so large that the assembler does not need to borrow. Our numerical results imply that as long as the assembler only borrow a small amount, the conclusion still holds.) Then, from Equations (A.4) and (B.17), $C(k_a(\mathbb{E}\omega + \gamma, y_a), q) = C(k_a(\lambda_a, y_a), q) = 0$, since $k_a(\mathbb{E}\omega + \gamma, y_a) = k_a(\lambda_a, y_a) = 0$. Consequently, the total supply chain profits under the pass-through contract is larger than that under the revenue-sharing contract.

Note that the above discussion does not rely on the value of $q$, as long as the assembler’s
working capital is small enough and assembler’s working capital is large enough. Particularly, let this $q$ be the assembler’s most preferred production quantity under the revenue sharing contract. Then, the inequality of (B.18) holds, and the pass-through contract pareto dominates the revenue sharing contract, i.e., both parties receive no smaller expected profit under the pass-through contract. Finally, note that under the pass-through contract, the assembler can even further optimize on $q$, under the constraint that $\Pi \geq \Pi^0$, and obtain even better profits.